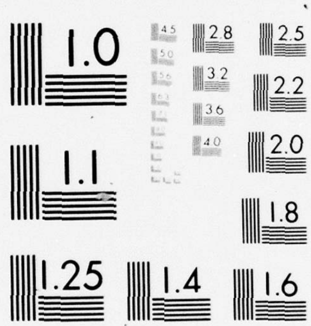
AD-A044 185 AIR FORCE INST OF TECH WRIGHT-PATTERSON AFB OHIO SCHO--ETC F/G 19/5
THE EFFECT OF RELEASE PARAMETER CORRELATIONS ON THE DISTRIBUTIO--ETC(U)
JUN 77 H A BROWN, M H CALLEN
UNCLASSIFIED AFIT-LSSR-25-77A NL

1 OF 3
ADA044185



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

AF INSTITUTE OF TECHNOLOGY

UNITED STATES AIR FORCE

AIR UNIVERSITY

AIR FORCE INSTITUTE OF TECHNOLOGY

Wright-Patterson Air Force Base, Ohio

3

THE EFFECT OF RELEASE PARAMETER
CORRELATIONS ON THE DISTRIBUTION
OF COMPUTER SIMULATED
BOMB IMPACTS

Harry A. Brown, GS-12
Monte H. Callen, Jr., Captain, USAF

LSSR 25-77A

The contents of the document are technically accurate, and no sensitive items, detrimental ideas, or deliterious information are contained therein. Furthermore, the views expressed in the document are those of the author and do not necessarily reflect the views of the School of Systems and Logistics, the Air University, the United States Air Force, or the Department of Defense.

USAF SCN 75-20B AFIT Control Number LSSR 25-77A

AFIT RESEARCH ASSESSMENT

The purpose of this questionnaire is to determine the potential for current and future applications of AFIT thesis research. Please return completed questionnaires to: AFIT/SLGR (Thesis Feedback), Wright-Patterson AFB, Ohio 45433.

1. Did this research contribute to a current Air Force project?

   a. Yes b. No

2. Do you believe this research topic is significant enough that it would have been researched (or contracted) by your organization or another agency if AFIT had not researched it?

   a. Yes b. No

3. The benefits of AFIT research can often be expressed by the equivalent value that your agency received by virtue of AFIT performing the research. Can you estimate what this research would have cost if it had been accomplished under contract or if it had been done in-house in terms of manpower and/or dollars?

   a. Man-years ________ $ ________ (Contract).

   b. Man-years ________ $ ________ (In-house).

4. Often it is not possible to attach equivalent dollar values to research, although the results of the research may, in fact, be important. Whether or not you were able to establish an equivalent value for this research (3 above), what is your estimate of its significance?

   a. Highly Significant b. Significant c. Slightly Significant d. Of No Significance

5. Comments:

________________________ ________________________
Name and Grade Position

________________________ ________________________
Organization Location

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER (14) AFIT-LSSR-25-77A	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) (6) THE EFFECT OF RELEASE PARAMETER CORRELATIONS ON THE DISTRIBUTION OF COMPUTER SIMULATED BOMB IMPACTS.		5. TYPE OF REPORT & PERIOD COVERED (9) Master's Thesis.
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) (10) Harry A. Brown, GS-12; Monte H. Callen, Jr., Captain, USAF		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Graduate Education Division, School of Systems and Logistics, Air Force Institute of Technology, WPAFB, OH		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Department of Research and Administrative Management (LSGR), AFIT/LSGR, WPAFB, OH 45433		12. REPORT DATE (11) June 1977
		13. NUMBER OF PAGES 185 (12) 203p.
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

APPROVED FOR PUBLIC RELEASE AFR 190-17.

JERRAL F. GUESS, CAPT, USAF
Director of Information

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

BOMBING
COMPUTER MODEL
BOMB IMPACT DISTRIBUTION
JOINT MUNITIONS EFFECTIVENESS MANUALS
BIVARIATE NORMAL

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Thesis Chairman: Edward J. Fisher, Lieutenant Colonel, USAF

012250

DD FORM 1473, 1 JAN 73 — EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

This study is the continuation of an ongoing effort at the School of Systems and Logistics to build a general bombing model. Using as a basis the Downs and Forseth computer model which incorporates the six release parameters altitude, true airspeed, flight path angle, heading, and lead/trail, the authors sought to make certain modifications which would more closely approximate real world bombing results. The Downs and Forseth model generated bomb impacts based on the assumption that the six release parameters were independent one from all others. The authors challenged the validity of this assumption and asserted instead that pilots make coordinated control inputs which improve bombing accuracy. This study used data recorded during the bombing accuracy evaluations of the A-10 aircraft as the basis for establishing correlations between various pairs of release parameters. It provides statistical analysis and graphs of the resultant bomb distributions. Results indicate that bomb impact distributions generated from correlated release parameters are grouped more closely around the desired target than bomb impact distributions generated from independent release parameters.

LSSR 25-77A

THE EFFECT OF RELEASE PARAMETER CORRELATIONS ON THE DISTRIBUTION OF COMPUTER SIMULATED BOMB IMPACTS

A Thesis

Presented to the Faculty of the School of Systems and Logistics

of the Air Force Institute of Technology

Air University

In Partial Fulfillment of the Requirements for the

Degrees of Master of Science in Logistics Management

and Master of Science in Facilities Management

By

Harry A. Brown, BSAE	Monte H. Callen, Jr., BSCE
GS-12	Captain, USAF

June 1977

Approved for public release;
distribution unlimited

This thesis, written by

Mr. Harry A. Brown

and

Captain Monte H. Callen, Jr.

and approved in an oral examination, has been accepted by the undersigned on behalf of the faculty of the School of Systems and Logistics in partial fulfillment of the requirements for the degrees of

MASTER OF SCIENCE IN LOGISTICS MANAGEMENT
(Mr. Harry A. Brown)

MASTER OF SCIENCE IN FACILITIES MANAGEMENT
(Captain Monte H. Callen, Jr.)

DATE: 15 June 1977

COMMITTEE CHAIRMAN

ACKNOWLEDGEMENTS

Several people whose names would not otherwise appear in this document deserve recognition for significant contributions to those praiseworthy portions of this thesis. For those portions that are less than praiseworthy, we stand completely responsible.

First, we wish to acknowledge the extensive contributions by David B. Downs and Rolfe E. Forseth. Without their development of the bombing model, we would not have had a vehicle with which to perform our investigation.

In addition, we wish to thank Lieutenant Colonel Edward J. Fisher for serving as our thesis chairman. His patient indulgence and continuous support provided us the freedom to study the areas of our choosing. However, he was always available with professional guidance the many times that we strayed beyond our own limits.

We also express our thanks to Barbara Barnes, our typist, whose labor somehow transformed chaos into order. Her many hours of deciphering and typing made the final presentation of this thesis possible.

Last, but certainly not least, we wish to thank our wives, Carron and Norma, for their significant role in this effort. Without their love and understanding, we could not have given so much of ourselves to this research effort.

TABLE OF CONTENTS

Page

APPENDICES

LIST OF TABLES

Table		Page

LIST OF FIGURES

Figure Page

LIST OF SYMBOLS

AGL	Above ground level
CAS	Calibrated airspeed--ft/sec
FPA	Flight path angle relative to the horizontal--degrees
H_A	Alternative hypothesis
H_O	Null hypothesis
HDG	Heading--degrees
KCAS	Knots calibrated airspeed
KIAS	Knots indicated airspeed
MSL	Mean sea level
r	Pearson product-moment coefficient for sample
R^2	Coefficient of multiple determination
R_p	Perfect range value--ft
V_x	Velocity with respect to ground along X-axis--ft/sec
V_y	Velocity with respect to ground along Y-axis--ft/sec
V_z	Velocity with respect to Z-axis (vertical)--ft/sec
X	Distance along run-in-line--ft
$\overline{X}$	Mean value of range--ft
X_A	X-distance of aircraft at time of bomb release--ft
X_B	X-distance of bomb impact point from target--ft
Y	Distance off run-in-line (offset)--ft
Y_A	Y-distance of aircraft at time of bomb release--ft
Y_B	Y-distance of bomb impact point from target--ft
Z	Height above target--ft
Z_A	Z-distance of aircraft at time of bomb release--ft
ρ	Population coefficient of correlation

CHAPTER I

INTRODUCTION

STATEMENT OF THE PROBLEM

It has not been fully determined what effect pilot attempts to perform small corrections during the final phase of weapons delivery have on bombing accuracy. In the past, improvements in the accuracy of aerial weapons delivery have been accomplished either by improving the weapons themselves or by developing more sophisticated avionics which make it easier for the pilot to predict weapons impact points (13:1). The general assumption has been that if the aircraft meets all specified handling qualities, the pilot could accurately bomb a target (13:1). Under this criterion, very little attention has been given to the contributions of pilot error (deviations from the desired bombing maneuver) on bomb impact accuracy (13:1). Therefore, the need for a study of the relationship between pilot corrections and bombing accuracy was indicated.

JUSTIFICATION

Bombing accuracy continues to be an area of primary concern to the Air Force. For example, one primary mission of the Strategic Air Command (SAC) is to drop bombs on enemy

targets. Perfect crew performance enroute to the target is negated if the target is missed (4:2). The procedure which a SAC crew follows while attempting to drop bombs on a target can be visualized as a series of inputs (control corrections) and outputs (bomb impacts) (4:44). As such, it is subject to analysis to determine which combinations of inputs produce the optimum output (4:44). This same concept can be applied to the development, production, and implementation of aircraft which have bombing and close air support as primary missions.

For aerial weapons, the specific need for bombing accuracy occurs in three areas: (1) in the context of conventional capabilities, (2) aircraft survival, and (3) propaganda potential (8:2). The first of these simply means that a conventional bomb, to be effective, must be dropped on or close to the desired target. In the area of aircraft survival, a bombing aircraft is subjected to threats from enemy aircraft, surface-to-air missiles, and various forms of anti-aircraft cannon fire (1:39). Accurate bombing negates the requirement for additional airstrikes and therefore reduces the threats against aircraft survival. Lastly, off target bombs may strike non-military or friendly positions. The psychological effects of these misses are a source of propaganda for the enemy.

Bombing accuracy is affected by a number of factors. Among the more important causes of bombing inaccuracy are

(1) the effect of wind, (2) pilot error, and (3) errors of the on-board computer (21:4). Depending on weather, aircraft capabilities, weapon type, and enemy defense environment, various attack modes are used for bomb delivery. The pilot is required to make necessary corrections during the course of bomb delivery. The errors introduced by the pilot as a result of these corrections are relatively independent of system errors. He is attempting to return his aircraft to the desired flight parameters. The same computational methods used for determining the effects of avionic systems errors may be applied in the evaluation of effects caused by pilot error (21:4). However, insufficient data limits this type approach to the problem (21:4). This is a significant point because,

> . . . the effect of pilot error is probably greater than that caused by the on-board computer system. In any case, experimental data derived from use of a simulator should be obtained in order to substantiate this assertion [21:4].

Still, some research has been done.

Robert B. Rankin developed a model which attempted to account for the cross-correlation of error sources due to pilot correction of one variable error with intentional deviation in another (13:1-2). Through further research based on this Rankin study, Hovde concluded that F-4C pilots could minimize range error (bomb impacts long or short of the target) by concentrating on airspeed control rather than altitude control when they were approximately on the desired

dive angle (13:68). However, he did not investigate pilot contributions to deflection errors (impact errors to the left or right of the target). He concluded that simulation studies are needed to provide a better basis for the prediction of impact errors resulting from various combinations of control inputs (13:68).

New aircraft such as the A-10 contain sophisticated avionics packages designed to improve the interface of man and machine as a means for obtaining bombing accuracy. The primary obstacle to developing such a system is the lack of knowledge of the performance characteristics of the pilot; that is, an inability to construct a mathematical model of the pilot (12:1). A well trained pilot will select the optimum control or mix of controls to accomplish his mission. Through experience in a particular aircraft, he has learned exactly the magnitude and phasing of control inputs required to successfully deliver a bomb on target (12:18). However, at the present time, this information is not readily available in a usable format.

The development of a correlation between pilot corrections for bomb release deviations and bombing accuracy would provide a basis for a better understanding and application of modern attack capabilities (11:58). Based on the Downs and Forseth model, the effects of pilot correction on the subsequent bomb impact distribution could be determined by exploring the possibility of some correlation between the

computer model bomb release parameters (8:87). In addition, the exploration of "real world" correlation parameters and their comparison with computer model produced data would form a basis for the analysis of impact distributions and bombing accuracy (8:87).

BACKGROUND

We conducted a generalized topical search in an effort to locate any treatises on bombing effectiveness and, more specifically, error analysis computer programs that might be relevant to the proposed research. A review of the available literature revealed limited recent research activity within the realm of error prediction for aircraft weapons delivery systems. However, those studies that were examined could be divided into two basic categories of investigation. One category concerned investigations into the authenticity of the assumption that bomb impacts around a target are bivariate normally distributed (23:1). The other category regarded the determination of errors resulting from variations of attack maneuver parameters, and avionics instrumentation and electronic equipment parameters (24:1-1). Although both areas of research are separate and distinct, they are complementary in that the assumed impact distribution constitutes the basis from which bomb plans and tactics are developed (23:1). Therefore, bombing plans and tactics dictate the equipment design and attack maneuvers required for weapons delivery.

Investigations of the assumption that bomb impacts around a target are bivariate normally distributed have provided some basis for questioning the validity of this assumption. An analysis of empirical bombing data by Chamberlain revealed that range and deflection errors were independent, but not normally distributed when altitude, velocity, pitch, and type of target were varied (5:1-5).

Berry and Laugginger questioned the normal distribution assumption from a theoretical perspective by generating bomb impacts around a target using computer simulation techniques. They employed a computer model developed by Mr. Jack Watts of General Dynamics Corporation. Watts' model had the capability of evaluating the effects of varying altitude, velocity, pitch, and heading (3:30). Berry and Laugginger analyzed six weapon release profiles. These profiles were considered to be the "typical" weapons delivery maneuvers which aircraft perform in order to direct their bombs toward a target. These profiles were basically classified according to three combinations of release point characteristics: low or high altitude, low or high drag bomb, and level or dive attitude (3:16). Impact distributions were generated for each of the profiles by holding one of the model release parameters (altitude, velocity, pitch, and heading) constant while varying the other three parameters normally. One exception to this normal variation was a case where airspeed was varied according to a Beta distribution. The resultant

distributions were determined to be bivariate normally distributed except for the high drag bomb when airspeed was distributed according to a Beta probability distribution (3:68-78). This very interesting failure of bomb impacts to be normally distributed when one of the release parameters was not varied normally was pursued by Downs and Forseth in a follow-on study.

Downs and Forseth Model

Downs and Forseth extensively modified the Berry and Laugginger computer model to account for six weapon release parameters: airspeed, altitude, pitch, lead or trail, heading, and offset (8:30). The relevance and import of these parameters on bombing accuracy can be demonstrated by consideration of the concept of the perfect bomb release point.

This point is defined to be that single point in three-dimensional space at which a bomb must be released (set free of all external forces except gravity, aerodynamic forces, and atmospheric generated forces) so that it may follow a ballistic trajectory to a precise impact upon the designated target point (8:16). The release parameters are those initial free-flight conditions which determine if the actual bomb release point coincides with the perfect bomb release point. The variation of only one actual release parameter from the ideal parameter will cause the bomb to miss

the target point. The bomb could be released above or below (altitude error), forward or behind (lead or trail error), or to the left or right (offset error) of the perfect release point. The initial release could occur at a velocity faster or slower (airspeed error) than the nominal. The centerline or longitudinal axis of the bomb could have some angle of incidence to the relative wind direction (pitch error) measured in a vertical plane passing through the center of the bomb. Also, the direction of the velocity of the bomb at release, measured in a horizontal plane, could be at some angle (heading error) to the left or right of the target. This angle is measured relative to a vertical reference plane that passes through the perfect release point and the target. If a departure from the perfect release point causes the bomb to either fall short of or beyond the target, it is called range error. Deflection error occurs if a bomb falls to the left or right of the target (8:17-9).

The parameters mentioned above were varied from the perfect release parameters by Downs and Forseth to ascertain the effect on the bomb impact distributions. Several classes of variations were studied, two of which are most pertinent to this investigation. The first class of interest varied one parameter normally while the other five were held constant. Only those distributions resulting from variations of offset and lead or trail parameters were found to be normally distributed. Results from variations of heading were

inconclusive (8:65). The second pertinent portion of their study was the re-analysis of the same six weapon release profiles which Berry and Laugginger had studied, plus one profile determined by Downs and Forseth to be "typical." This "typical" profile had been determined from informal discussions with their pilot classmates (8:58-9). All input parameters to these profiles were generated according to a normal probability distribution except as noted in profiles two and three. A brief description of the profiles and their test results (8:59-64) are as follows:

1. High altitude, straight and level release of a low drag bomb--deflection errors were normally distributed while range errors were not.

2. Beta distributed airspeed, high altitude, straight and level release of low drag bomb--deflection errors were normally distributed while range errors were not.

3. Gamma distributed offset, high altitude, straight and level release of low drag bomb--range and deflection errors were not distributed normally.

4. Low altitude, straight and level release of low drag bomb--range and deflection errors were not distributed normally.

5. Low altitude, dive release of low drag bomb--distribution was bivariate normal.

6. Low altitude, straight and level release of high drag bomb--range and deflection errors were not distributed normally.

7. Low altitude, shallow dive release of high drag bomb--range and deflection errors were not distributed normally.

The low altitude, dive release of a low drag bomb was the only case that was bivariate normally distributed. Therefore, results of six of the seven samples supported the Downs and Forseth hypothesis that bomb impacts around a target (as generated by their model) were not bivariate normally distributed.

Dynamic Weapon Delivery Analysis Program

While a number of researchers have been investigating bomb impact distributions around a target, few have attempted to predict the performance and error of specific weapon delivery systems. However, a computer program designated as the Dynamic Weapon Delivery Analysis program was recently produced for the Naval Air Systems Command by Autonetics, North American Rockwell. This extensive program was developed to assess weapon delivery accuracy under a variety of dynamic bombing maneuvers for a wide range of possible avionics instrumentation, and designation and attack procedures (24:1-1).

The program consists of seven computational stages which permit a thorough analysis to be conducted to pinpoint the critical sources of error within the overall system or specific subsystems. The first stage performs computation of the ballistic trajectory data for a bomb released under

any one of three chosen attack maneuvers: vertical plane attacks (level, dive, toss, etc.), lateral attack using a Synthetic Array Radar system, or a generated dynamic attack maneuver. The second stage computes the variation of impacts due to system and weapon errors. Stage three computes the total error propagation from release source to impact for four error groups: avionics, non-avionics, biases, and random errors. This summation is based on the assumption that all error propagations at impact form a bivariate Gaussian error pattern (i.e., bivariate normally distributed). Stage four computes the subgroup errors which permit the evaluation of performance of individual instruments within the context of the total system performance. Some typical subgroups are: inertial navigation errors, radar errors, and air data subsystem errors. Stage five then computes the general impact pattern error. The pay-off ratio is computed in stage six so that the dominant error sources can be determined. This pay-off ratio is the percentage variation of the impact error due to a one percent variation of a system parameter. This pinpoints those subsystems from which the greatest amount of accuracy improvement could be derived if the system were redesigned to increase performance. Finally, stage seven computes the probability of kill for the bomb impacts generated (24:1-2 thru 1-4).

The preceding discussions illustrate the extensive studies and research attempting to determine the distribution

of bomb impacts around a target for different variations of release parameters. Also, extensive computational tools have been developed to analyze the effects of variations in attack maneuvers, release conditions, designation procedures, weapons, and instrument and target tracking subsystems. However, these studies have all ignored one essential factor in the total bombing system: the man in control. Only in those instances where the pilot/bombardier is excluded from the control-loop by automatic weapon delivery devices is it acceptable to ignore the effects of human error on bomb impact distributions (13:2).

The literature review offers insight into what methods are available for investigating pilot error effects on bomb impact distributions. The Downs and Forseth model was found to be most applicable because it permits variations of pilot controlled parameters. Additionally, the profiles investigated by Downs and Forseth provided a basis for comparison of impact distributions generated through correlated input parameters varied in much the same manner as a pilot's reactions. We have extended the analysis to determine what pilot controlled release parameters should be most closely monitored to improve weapon delivery effectiveness.

OBJECTIVES

The objectives of our research were to:

1. Evaluate actual bombing data to determine if the impacts were bivariate normally distributed. The data that we analyzed was the results of the A-10 Bombing Accuracy Demonstration conducted in the Fall of 1975.

2. Use the A-10 bombing data to determine what correlations exist between the release parameters for that particular data sample.

3. Modify the Downs and Forseth model to include the capability of correlating release parameters. The model will be capable of accepting a variance-covariance matrix for the following six release parameters: airspeed, altitude, pitch, lead or trail, heading, and offset.

4. Determine if attempted pilot corrections during the final phase of weapons delivery improves bombing accuracy.

RESEARCH HYPOTHESIS

The variance of computer generated bomb impact distributions resulting from variation of correlated release parameters is less than the variance of those distributions resulting from independent release parameters.

CHAPTER II

METHODOLOGY

INTRODUCTION

The purpose of this chapter is to specify the methods we used for data analysis. The analysis method for each research objective was examined in the order listed in the previous chapter. The following topics will be discussed:

1. The data source, methods of collection, and reasons for choosing the data source.

2. The methods used to analyze the distribution of the release parameters and bomb impacts. This includes a brief summary of the computer program that was used.

3. A description of the correlation analysis of release parameters which provided the variance-covariance matrices.

4. The Downs and Forseth computer model. This includes a discussion of the model modifications required to incorporate the variance-covariance matrices.

5. The statistical tests required for support of the research hypotheses. The appropriateness of these tests will then be justified.

A-10 DATA

The theoretical investigation of the effects of bomb release parameter correlations was based on actual pilot performance during bombing operations in a training environment. A population of release parameters and bomb impacts was collected during the A-10 Bombing Accuracy Demonstration Tests. These tests were required by government contract to demonstrate the bombing accuracy of the A-10 aircraft. The data was collected during the period from September 24 to October 25, 1975. The tests were conducted at the Edwards AFB bombing range (2000 feet mean sea level altitude) and were witnessed by both Fairchild Republic Company and USAF/ ASD personnel (10:11-12).

Eight bombing profiles were flown for which the bomb release parameters and bomb impacts were recorded. Four Air Force evaluation pilots flew each profile nine times (total of nine drops per profile per pilot). The X, Y, and Z coordinates of the aircraft space position and the associated components of velocity at the time of release were the parameters recorded. These values were determined with respect to the bombing target coordinate system using photo theodolite and radar tracking. The center of the target was the origin of the coordinate system. The X-axis was the approach direction or run-in line of the aircraft and was measured positive downrange from the target. The Y-axis was perpendicular (cross range) to the X-axis and was measured positive

when the release or impact was to the left of the target. The Z-axis was the vertical direction (altitude) and was positive upward. The target altitude was approximately two thousand feet above sea level. The bomb impacts were recorded for each set of release parameters. The impact points were recorded by surveying the X and Y distances of the impacts from the target (10:13-4). All data were recorded at the ratio level[1] which permits the comparison of observations with respect to order, distance, and a unique origin (zero).

The sample of release parameters consists of all observations recorded for pilots two and four for all eight profiles. This sample was chosen because data for the eight profiles was not available for pilots one and three.

This data source was chosen as a basis for the proposed research for four reasons. The first reason was because of the availability of the data. Since time, money, and facilities were not available to obtain precise bombing data for this research effort, previously conducted test results obtained from ASD seemed most appropriate. The second reason for the choice was because the A-10 aircraft was designed for the primary mission of close air support of ground operations. The possibility of generalizing some results from this research

[1]Ratio level data results from the measurement of physical dimensions such as weight, height, distance and area.

effort to improve mission effectiveness made the selection extremely attractive. Due to the limited number of samples, only subjective generalizations are possible. Results from this research effort can only provide general guidelines for improving pilot techniques during bombing deliveries made from the A-10 aircraft.

The third reason for choosing this data source was because the Downs and Forseth computer model does not account for the effects of wind on bomb impact distributions. One of the ground rules for the A-10 bombing tests was that no drops would be made when bombing mission pattern winds exceeded twenty knots (10:27). This provision reduced modeling error produced by not accounting for wind effects within the model.

The fourth reason for choosing this data source was because the release parameters had been recorded for each impact. The recorded release parameters are the same parameters that are required by the model to generate theoretical impacts. Generation of the theoretical distributions using the observed release parameters permitted the comparison of the empirical and theoretical distributions of impacts. This comparison revealed the degree to which the computer model is capable of predicting the real world situation.

TEST FOR NORMALCY

Downs and Forseth were unable to support their research hypothesis that bomb impacts are bivariate normally

distributed (8:79). They used the actual mean and standard deviation of the observed release parameters in a sample of twenty-seven A-10 bomb drops. Values for the heading parameter were subjectively chosen since the actual values were not in recognizable form. Based on these occurrences, further tests of the "real world" data were required to determine whether or not the bomb impacts were bivariate normally distributed. Heading parameters were calculated and included in this analysis.

The distribution of bomb impacts around a target forms a bivariate distribution with the X and Y coordinates of the impact points as the random variables (8:42). The X value (range error or ordinate) is the number of feet long (+) or short (-) of the target that the bomb falls. Similarly, the Y value (cross range or abscissa) is the number of feet to the right (+) or left (-) of the target that the bomb falls (this coordinate system is opposite to that of the A-10 data, but is consistent with the Downs and Forseth model) (17:18 as referenced in 8:41). The distribution is bivariate normal if the following conditions are met:

1. X and Y are independently distributed,
2. the distribution of X is normal,
3. the distribution of Y is normal (25:237-251).

The model subprograms SINDEP and SSIMFIT were used to perform these tests.[2]

---

[2]For a listing and indepth discussion of the subprograms SINDEP and SSIMFIT refer to Downs and Forseth thesis, SLSR 18-76A, June 1976.

SINDEP is the independence test program which uses the Chi-Square Goodness of Fit Test (8:167). Wonnacott and Wonnacott's "Contingency Test for Independence" is the basis for the program and has the following hypothesis, which is the same as that used by Downs and Forseth:

$$H_O: \quad P(X|Y) = P(X), \quad P(Y|X) = P(Y),$$

$$H_A: \quad P(X|Y) \neq P(X), \quad P(Y|X) \neq P(Y),$$

where X and Y are events of interest. As a test of independence, chi-square is not a measure of the degree of form of relationship but only a help in determining if the relationship between X and Y is significant (6:376). The null hypothesis (H_O) states that X and Y are statistically independent of each other. The alternate hypothesis (H_A) states that X and Y are not independent. This test compares

$$X^2_{sample} = \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$

where:

1. The test is based upon data grouped into cells,
2. n is the number of cells,
3. E_{ij} must meet the following conditions:

   all $E_{ij} \geq 1$

   at least 80% $E_{ij} \geq 5$,

4. O_{ij} = observed frequency in cell ij,

E_{ij} = expected frequency in cell ij.

with:

$\chi^2_{critical}$ (a tabular value)

where:

1. $\alpha = .05$, the acceptable risk level,

2. degrees of freedom = (no. of rows -1) x (no. of columns -1) (8:43-4).

An alpha level of 5% means that there is a 5% probability of making an incorrect decision, i.e., rejecting a null hypothesis when it is actually true (6:322). The null hypothesis is supported at the 95% confidence level if

$$\chi^2_{critical} > \chi^2_{sample} \quad (3:35).$$

This means that there is insufficient evidence to refute the hypothesis that events X and Y are independent.

SSIMFIT is the subprogram which tests X and Y to determine if their respective distributions are normal. It uses a Chi-Square Goodness of Fit Test which is a valid test when a distribution is either discrete or continuous and the parameters of the population are unknown. In the case of the A-10 bomb drop data, the population parameters of the bomb impact distributions were not known conclusively. Downs and Forseth originally used the Kolmogorov-Smirnov (K-S) goodness of fit test as a test for normalcy (16:1). Later, they

changed the SSIMFIT program to include the Lilliefors test for normalcy.

The Lilliefors test is a modified K-S test which does not require that the population parameters be known (7:307). The difference between K-S and Lilliefors is the respective critical values. For a 95% confidence interval and with $n > 35$, the K-S critical value is $1.36/\sqrt{n}$ and the Lilliefors critical values for $n > 30$ is $.886/\sqrt{n}$ (16:Appendix). SSIMFIT performs the K-S test before it performs the chi-square test for normalcy (8:82). Downs and Forseth discovered that by substituting a Lilliefors critical value for the computed K-S value, they obtained a more powerful test (8:81-2). For values of $n > 35$, the Lilliefors critical value is used in the SSIMFIT test for normalcy.

SSIMFIT uses the following Chi-Square Goodness of Fit hypothesis in the test for normalcy:

$$H_O: \; X \sim \text{normal}$$

$$H_A: \; X \nsim \text{normal}$$

This test compares:

$$\chi^2_{\text{sample}} = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i} ,$$

where:

1. n = the number of cells
2. O_i = observed frequency in cell i,
3. E_i = expected frequency in cell i,

with:

$$\chi^2_{\text{critical}} \quad \text{(a tabular value)}$$

where:

1. $\alpha = .05$, the acceptable risk level,
2. degrees of freedom = (no. of cells -1) x (no. of estimated parameters) (8:44-5).

The null hypothesis is supported at the 95% level if

$$\chi^2_{\text{critical}} > \chi^2_{\text{sample}} \qquad (3:37).$$

By accepting the null hypothesis, we may infer that there is insufficient evidence to conclude there is a significant difference between the sample distribution and a normal distribution (i.e., the sample distribution approximates a normal distribution). If the null hypothesis is rejected, the alternative hypothesis is accepted. In this case, we infer that the sample distribution does not approximate a normal distribution.

CORRELATION OF RELEASE PARAMETERS

An underlying assumption for this research effort is that pilots perform interrelated aircraft control corrections prior to bomb release to improve accuracy. In order to determine what relationships existed between control corrections, the bomb release data was analyzed for correlation.

The principal assumption for correlation analysis is that the variables (release parameters) are bivariate normally distributed (14:7). To meet this assumption, all release parameters must be both independent and normally distributed. To conduct correlation analysis, the parameters were assumed to be independent. The assumption of normalcy for each release parameter was tested to insure that generating random observations from a multivariate normal distribution was appropriate for the "real world" distributions. The tests for normalcy used the same methodology outlined for SSIMFIT in the preceding section. However, the AFLC CREATE computer timesharing library version of SIMFIT was used for the analysis.[3]

All release parameters were subjected to a bivariate correlation analysis. This analysis was performed to obtain a measure of the degree of association between each pair of release parameters. The resulting coefficients of correlation provided an indication of the relative strength of the linear relation[4] between each combination of pairwise groupings of release parameters. The summary statistic which was used to measure the strength of this linear relation was the Pearson product-moment coefficient ($\underline{r}$). The coefficient

---

[3]For a complete discussion of SIMFIT program see "Users Guide for SIMFIT" prepared by William E. Glaesemann, October 1975.

[4]If one variable is fixed at a specified value, the other may be predicted on the basis of its linear relationship to the fixed variable.

r is a sample-based estimate of the population coefficient of correlation ρ (9:402-4). The correlation coefficient for each grouping, say x and y (ρ_{xy}), was subjected to the following hypothesis test to determine statistical significance at the 95% confidence level (14:5):

$$H_o: \rho_{xy} = 0$$
$$H_A: \rho_{xy} \neq 0$$

If the null hypothesis could be rejected, we could conclude that a statistically significant, linear relation existed between the pairwise grouping of release parameters. The subprogram PEARSON CORR of the Statistical Package for the Social Sciences (SPSS) computer programs was used to perform the correlation analyses. This subprogram provided the Pearson product-moment correlation coefficients and their corresponding levels of statistical significance for each pairwise grouping of release parameters. This analysis also provided the variance-covariance matrices which were used to generate correlated, random observations of release parameters in the bombing model.

MODEL DESCRIPTION

Based upon the prior work of Chamberlain (1971), Richardson (1971), and Berry and Laugginger (1975), Downs and Forseth (1976) developed a computer model which simulates bomb impact errors with respect to six release parameters:

airspeed, altitude, pitch, lead or trail, heading, and offset. All other factors in the model that could contribute to range or deflection error are set to zero or held constant (8:30). By repeating the process of simulating a bomb's fall and plotting its impact relative to a target, sample bomb impact distributions are generated (8:31). The model produces impact errors by independently varying one or more of the release parameters (8:31).

The model is written in FORTRAN Y programming language. It accepts three types of parameters: control, program, and release (8:33). The model control parameters which can be varied are the following:

1. the type of bomb
2. the target latitude
3. the target altitude
4. bomb release equipment time delay
5. bomb ejection velocity
6. the number of bombs dropped (8:33).

One of three types of bombs can be selected: MK-106, BDU-33, A/B, or MK-82. The model accepts three program parameters: the random number generator seed value, the ballistic path computation iteration time, and the initial cell size for sorting the bomb impacts (8:33). Lastly, the six release parameters are entered.

Using control, program, and release parameters; atmospheric characteristics; and the bomb's weight, diameter, and drag coefficient, the model computes a ballistic path for the predicted time of fall, range, trail, and bomb

impact point (8:33). In a given sample the control and program parameters are constant for each bomb dropped. The model can also hold one or more release parameters constant.

Variability of a single release parameter or a combination of release parameters results in release parameter errors which cause impact errors around the desired target (8:34). The model writes all the impacts onto output files which can be used for statistical analysis or a graphical summary.

The complete Downs and Forseth model is composed of five programs, one subprogram, and five data files.[5]

Accuracy

Accuracy refers to how close the model simulates a perfect bomb drop. The single most significant factor affecting the model's accuracy is the iteration time (8:35). Iteration time is the constant interval of time between recalculation of the falling bomb's altitude, pitch, airspeed, Mach[6], acceleration, etc. (8:35).

Precision

The following procedures remain as instituted by Downs and Forseth in the computer model:

---

[5]For a detailed discussion of the Downs and Forseth model refer to their thesis, SLSR 18-76A, June 1976.

[6]The velocity of the bomb relative to the speed of sound at altitude.

1. All statistical tests will be performed at the 95% confidence level.

2. All impact point coordinates will be rounded to the nearest tenth of a foot.

3. The number of standard deviations allowed around a release parameter will be set at five. If a release parameter is normally distributed, five standard deviations will include 99.99997133% of all bomb impacts; less than one in one million impacts falls outside five standard deviations (18:972).

Assumptions

1. The model, both conceptually and mathematically, is assumed to be independent of aircraft type. However, bomb ejection velocity can be modeled.

2. The atmosphere is assumed to have the characteristics specified in the 1962 United States Standard Atmosphere. The standard atmosphere provides atmosphere temperature at all altitudes from -5 to 90 kilometers. Dynamic atmospheric forces, such as wind, were not considered.

3. All target altitudes were assumed to be two thousand feet mean sea level (MSL) (to correspond to conditions at Edwards AFB).

4. Each bomb was assumed to be "perfect" with respect to weight, dimensions, etc.

5. The error caused by rounding impact point coordinates to the nearest tenth of a foot is assumed to be negligible.

Limitations

1. The model is limited to bomb release altitudes of -16,404 feet (-5 kilometers) to 295,276 feet (90 kilometers) and zero winds (as a result of assumption 2 above).

2. Impacts generated by heading errors are accurate only within the range of ±10° heading error.

3. No parameters affecting "real world" bomb impact errors, other than the stated control and release parameters, were considered (8:40).

MODEL MODIFICATION

The Downs and Forseth bombing model was modified so that correlated release parameters could be used to generate simulated bomb impacts. This was accomplished by incorporating a FORTRAN subroutine which would randomly generate a set of release parameters from a multivariate normal population with a specified variance-covariance matrix. This subroutine was developed by Professor W. Earl Sasser of the Harvard Business School (19:397-8). The variance-covariance matrix for each profile was obtained as an optional statistic from the SPSS subprogram PEARSON CORR.

TESTS OF RESEARCH HYPOTHESIS

The following hypothesis test was used to test the research hypothesis:

$$H_0: \quad \sigma^2 x_{independent} \leq \sigma^2 x_{correlated}$$

$$\sigma^2 y_{independent} \leq \sigma^2 y_{correlated}$$

$$H_A: \quad \sigma^2 x_{independent} > \sigma^2 x_{correlated}$$

$$\sigma^2 y_{independent} > \sigma^2 y_{correlated}$$

The null hypothesis asserts that the variance of the sample with independent release parameters is less than or equal to the variance of the sample with correlated release parameters. The alternate hypothesis asserts that the variance of the independent sample is greater than the variance of the correlated sample.

The most powerful statistical tests are those which have the strongest or most extensive assumptions (22:19). An F-test, which has the following strong assumptions, was used.

1. The observations are independent. That is, the selection of any one impact from the population for inclusion in the sample does not bias the chance of any other impact for inclusion.

2. The impacts were drawn from normally distributed populations.

3. The impacts were measured at the ratio level.

4. The populations of impacts for the correlated and the independent release parameters have equal variances (22:19).

The choice of the F-test was justified since tests showed that these conditions existed in the data. An alpha level of 5% was used, which was consistent with the other tests performed and the model program design.

The F-statistic for each sample was computed using the following formula:

$$F_s = F_{sample} = \frac{S_I^2}{S_C^2},$$

where:

1. $S_I^2 = \frac{\Sigma(x_i - \bar{x}_i)^2}{n_I - 1},$

2. $S_C^2 = \frac{\Sigma(x_j - \bar{x}_j)^2}{n_C - 1}.$

and:

1. S_I^2 = the sample variance of the independent sample.

2. S_C^2 = the sample variance of the correlated sample.

3. n_I = the number of observations in the independent sample.

4. n_C = the number of observations in the correlated sample (25:1).

The critical value for the hypothesis test was:

$F_C \equiv F_{critical}$ (a tabular value with degrees of freedom equal n_I-1 and n_C-1.)

The null hypothesis was rejected if the following condition occurred:

$$F_S > F_C \qquad (26:2).$$

If the null hypothesis was rejected, we concluded that the

variance of the distribution of the independently generated impacts was statistically greater than the variance of the distribution for correlated impacts. This would support our research hypothesis that the variance of bomb impact distributions generated by variation of correlated release parameters is less than the variance of those distributions generated from independent release parameters.

CHAPTER III

DATA COLLECTION AND RESULTS

INTRODUCTION

In this chapter, we present the results obtained from the analysis of the data collected during our research. The specific research objectives outlined at the end of Chapter One each were examined using the methodologies set forth in Chapter Two. A list of the A-10 Bombing Accuracy Demonstration data obtained from the A-10 Systems Program Office at ASD is included as Appendix A. These data were used by the Air Force for bombing accuracy evaluations of the A-10. Four pilots participated in the evaluation. We analyzed only the data recorded for pilots numbered two and four for all eight profiles (profiles A-H). This sample was chosen because all data points for the eight profiles were not available for pilots numbered one and three.

This chapter is divided into four sections which correspond to our research objectives. First, we present results of the analysis of the A-10 bomb impacts recorded during the bombing accuracy evaluation tests. Our objective was to determine if those impacts were bivariate normally distributed. Secondly, we analyzed the release parameters from the bombing accuracy evaluation tests. Our objective was to determine if the six release parameters (heading,

altitude, offset, airspeed, pitch, and lead/trail) were normally distributed and, further, if any correlations existed between those parameters. Our third objective was to modify the Downs and Forseth model by adding a FORTRAN subroutine to generate release parameters from a multivariate normal population. A brief discussion of the incorporation of the subroutine is presented. Lastly, we present the results of our research hypothesis tests. Our objective was to show that the variance of bomb impact distributions generated from correlated release parameters is less than the variance of those distributions generated from independent release parameters.

ANALYSIS OF BOMB IMPACTS FOR NORMALCY

The bomb impacts recorded during the A-10 bombing accuracy evaluation tests were analyzed for independence and normalcy using the Downs and Forseth model programs, COLLAPSE, SINDEP, and SSIMFIT. The bomb impacts had been recorded as X and Y values of a two-dimensional coordinate system with the target located at the origin of the system (see Appendix A for a list of the bomb impact data). The X and Y coordinates for all impacts were written onto a data file for input to COLLAPSE. Since the number of actual impacts was considerably less than the number of computer generated impacts during a normal run of the Downs and Forseth model (144 versus 10,000), a slight modification of the COLLAPSE program was necessary. Essentially, only the reduction of

a counter was required. Specific changes made to the COLLAPSE program, by line number, are included as Appendix B.

The X and Y coordinates of the bomb impacts were tested for independence using the model program, SINDEP. The input file, INDEDATA, required by SINDEP was generated as a part of the COLLAPSE run. Results of the SINDEP analysis revealed that the X and Y impact coordinates were independent. The results of the tests for independence are shown in Appendix C.

The respective X and Y impact coordinate distributions were tested for normalcy using the model program, SSIMFIT. The input file, SIMDATA, used by SSIMFIT for accomplishing the statistical analysis was also generated as a part of the COLLAPSE run. Results of the SSIMFIT analysis showed that the distributions of the X and Y impacts for pilots numbered two and four (profiles A through H) were both normally distributed. The analysis results and histograms of the distributions are shown in Appendix D.

EVALUATION OF RELEASE PARAMETERS

Normalcy Tests

Our first objective was to determine if each of the six release parameter distributions were normal. The AFLC CREATE computer timesharing library version of SIMFIT was used for this analysis. This program was the basis for the Downs and Forseth SSIMFIT program. The only difference is

that SSIMFIT was modified to accept data grouped into cells from COLLAPSE. Each of three release parameters; heading, altitude, and pitch for profiles A through D, were individually tested. Likewise, these same parameters for profiles E through H were each tested for normalcy. The eight profiles were divided into these two categories because of the disparity between the nominal values of altitude and flight path angle for the A-D and E-H groupings (See Tables 3.1 and 3.2). The analysis showed that heading, altitude (except for profiles in the E-H grouping), and pitch each appear to follow a normal distribution. The remaining three release parameters; lead/trail, offset, and airspeed, were each tested separately using the combined data points for each parameter contained in profiles A through H. Each of these parameters also appear to follow a normal distribution. The results of the tests for normalcy are included in Appendix E.

Profile Statistics

Each common profile for pilots two and four (i.e., 2A and 4A, 2B and 4B, etc.) was combined for analysis to determine the statistics and parameter correlations for each of the eight individual profiles. These data combinations were analyzed using the SPSS subprogram PEARSON CORR to compute the Pearson product-moment correlations for each combination of pairs of parameters (See Appendix F for SPSS

TABLE 3.1

PROFILE RELEASE PARAMETERS

Profile	A	B	C	D	E	F	G	H
Release Altitude-Feet AGL	1800	1800	1800	1800	2500	2500	2500	2500
Release Airspeed-KCAS	260	280	300	320	280	300	320	340
Dive Angle-Degrees	30	30	30	30	45	45	45	45

TABLE 3.2

RELEASE PARAMETER TOLERANCES

	Profiles A-D	Profiles E-H
Nominal Dive Angle	30°	45°
Airspeed Tolerance	+20 KIAS -10	+25 KIAS -15
Altitude Tolerance	±500 Ft.	±500 Ft.
Dive Angle Tolerance	±5 Deg.	±5 Deg.

program). A two-tailed test was used since we did not have an explicit hypothesis concerning the direction of the respective coefficients (2:284). From the PEARSON CORR computer run, we were able to determine the pairs of release parameters which were statistically, significantly correlated ($\alpha = 0.05$). We also used a statistics option with the PEARSON CORR subprogram which provided the mean, variance, and covariance for each pair of release parameters. Using this information, we constructed a variance-covariance matrix for each profile to be used in the model to generate correlated impacts. Copies of these matrices are included as Appendix G. Based on the results of the PEARSON CORR computer run, the pairs of release parameters which were not statistically, significantly correlated were entered as a value of zero in the variance-covariance matrix. Based on the bivariate normalcy assumption, this, in effect, reflects independence (statistically speaking) between the respective pairs of release parameters.

Input File

The statistics for each profile grouping obtained from the PEARSON CORR analysis were inserted into the input file, DATAN5.5. The input file is the same as the Downs and Forseth input with the exception of lines 10 and 110-160. The mean for the appropriate release parameter is the first entry on lines 110-160. The variance-covariance values for

the respective release parameters are then entered in the next six fields. The resulting input file structure is a column vector of the six release parameter means followed by the 6X6 variance-covariance matrix. The input data file name "MVN" is entered on line 10 to generate correlated release parameters. The input data file name causes the program logic to read the input data file using the appropriate format and generate either independent (when DATAN5.5 is entered) or correlated release parameters (when MVN is entered). See Appendix H for a graphic explanation of the input file.

One minor difficulty was encountered in obtaining the values to be used in the input data files. The model uses the parameter lead/trail rather than range. The A-10 data provided the range of the target at release rather than the value for lead/trail with respect to the perfect release point. Consequently, we computed the perfect release point range by running the model with the appropriate input release parameters, zero lead/trail, and zero number of desired bombs. The model logic provides for one drop to be made in addition to the requested number of impacts. This computed impact point for the "perfect" bomb is then defined as the target. This computed value is output from the bombing model as range. The value output as trail is the aircraft position with respect to the target at bomb impact. Using this value for "perfect" range (R_p) and the A-10 mean data value for

range ($\overline{X}$), we calculated lead/trail using the following formula:

$$\text{Lead/trail} = R_p - \overline{X}.$$

These values are included in Table 3.3.

MODIFICATION OF THE DOWNS AND FORSETH MODEL

Incorporation of the Multivariate Normal FORTRAN subroutine into the Downs and Forseth model was accomplished only after extensive testing to insure that the subroutine was performing properly and producing accurate results. The actual test consisted of generating a population of one hundred observations for the six release parameters using the FORTRAN subroutine with the six-by-six (6X6) variance-covariance matrix for profile B. These one hundred generated observations were then analyzed using the SPSS subprogram PEARSON CORR. The values for the mean, variance, and covariance for the generated population were compared to the input values. The statistics for the generated population compared quite well with the input sample statistics. Comparison of the results are shown in Appendix I.

The subroutine FORTRAN statements were also compared to the theoretical formulas upon which they were based to determine their accuracy (20:98). One discrepancy was noted in statement number 6950 of the subroutine. An additional test was performed to determine the effect of this discrepancy

TABLE 3.3

VALUES FOR LEAD/TRAIL RELEASE PARAMETER

Profile	Range From STHESIS (R_p)	Range From A-10 Data ($\overline{X}$)	Lead/Trail
A	2010	1951.000	59.000
B	2144	2135.167	8.833
C	1993	1931.444	61.556
D	1942	1851.889	90.111
E	2052	1962.389	89.611
F	2046	2007.611	38.389
G	1890	1865.333	24.667
H	1806	1748.611	57.389

on the results. A discussion of the test and its results are presented in Appendix I.

A copy of the modified STHESIS program is contained in Appendix J. A brief explanation of the modifications, by line number, are also included.

TEST OF RESEARCH HYPOTHESIS

Prior to making any modifications to the Downs and Forseth model, we verified that it was performing the same as it had been during their research effort. We ran Case 4.4f (Low Altitude--High Drag Bomb--Straight and Level Release) from the Downs and Forseth thesis (8:64). We obtained the same results as they reported.

Having determined that the model was functioning properly, we ran each profile assuming that the release parameters were all independent. Next, we ran each profile using our modified version of the Downs and Forseth model which incorporated correlation between statistically significant pairs of release parameters. However, only two profiles, B and D, ran successfully. We determined that the multivariate normal subroutine could not generate release parameters for the other profiles due to the characteristics of the input data and the subroutine program logic. This fact provides for some rather interesting conclusions concerning the assumption of independence of the actual release parameters. These conclusions will be discussed in the following chapter.

The results of both the independent and correlated parameter runs for profiles B and D are included as Appendix K. We determined that the variance of the computer generated impacts resulting from correlated release parameters was considerably smaller than the variance of the impacts resulting from the independent release parameters. The results of the statistical tests for each of the hypothesis runs are also included in Appendix K.

CHAPTER IV

CONCLUSIONS, OBSERVATIONS, AND RECOMMENDATIONS

INTRODUCTION

In this chapter we provide a discussion on the results of our research. First, we present our conclusions concerning the data collected and recorded in Chapter Three. Second, we convey pertinent observations which we noted during the course of our study. Finally, we list recommendations for future study with suggested modifications to our existing bombing model.

CONCLUSIONS

Normalcy of Bomb Impacts

Based on our analysis, we concluded that the bomb impacts for pilots numbered two and four recorded during the A-10 Bombing Accuracy Evaluation tests were bivariate normally distributed. This conclusion was justified since the distributions of the X and Y impact coordinates were both normally distributed and independent. Thus, for our sample data, the JMEM assumption that "real world" bomb impacts follow a bivariate normal distribution was supported.

Release Parameters

As a result of the analysis, we concluded that each of the six release parameter distributions was normal. There was one discrepancy, however. The distribution of the altitude release parameter for the combined profiles E through H was not normally distributed.

Achievement of a normal distribution of release parameters should not be confused with superior pilot performance. In fact, repetitive duplication of release parameters would be the most desirable if the release values were close to the nominal values. This performance would produce a histogram with a central spike in the distribution located at the desired (nominal) release value. This, in fact, was the case for this combination of profiles E-H. Inspection of the histogram showed that the mode cell had approximately thirty-seven observations in it while the next most frequent cell had approximately twenty-one observations (see histogram in Appendix E). This could possibly lead to the conclusion that pilots more readily duplicate their release altitude performance during bombing at higher altitudes. This supports the intuitive belief that pilots pay more attention to achieving the nominal release altitude value when the ground is not so close.

Our assumption of independence between the release parameter distributions was not totally substantiated. As indicated in the previous chapter, only profiles B and D ran

successfully. A population of random multivariate normally distributed release parameters could not be generated for the other profiles using the Multivariate Normal FORTRAN subroutine.

The theoretical basis for the subroutine is a theorem

> . . . which states that if z is a standard normal vector, i.e., it contains independent normal variable components with zero mean and unit variance, there exists a unique lower triangular matrix C such that
>
> $$x = Cz + \mu.$$
>
> In this case $(x - \mu)$ has the variance-covariance matrix $V = C \cdot C'$ [20:98].

The subroutine logic uses the "square root method" to obtain the C matrix from the V matrix (variance-covariance matrix)[1]. We determined that the subroutine stopped as the result of an attempt to take the square root of a negative value to obtain a diagonal element of the C matrix (lines 6840-6870, Appendix J). This evidently occurred as the result of excessive multicollinearity, or the lack of independence, between the parameters.

A lack of independence would be indicated when the sum of the squares of the correlation coefficients for one variable exceeded one. The coefficient of determination, r^2, ". . . is a measure of the proportion of variance in one variable 'explained' by the other (2:279)." We examined the

---

[1]C' is the transpose of the C matrix; μ is the mean of the resulting distribution of x.

sum of the coefficients of determination for each row for profiles B and D. We found that all the row sums were less than or approximately equal to one. (Implications of the subroutine analyzing profile B with a row sum slightly in excess of one are discussed in Appendix I.) The profiles other than B and D all had row sums in excess of one. To examine this phenomenon, we reran profile A arbitrarily treating altitude and lead as uncorrelated (setting covariance equal to zero). This was the lesser significant correlation coefficient ($\alpha = 0.031$) in the row whose sum exceeded one. The modified variance-covariance matrix then successfully ran in the subroutine. We concluded that all the profiles could be made to perform, but this would be inconsistent with the interrelatedness among and between release parameters so evident in the A-10 data.

Since only profiles B and D ran in the FORTRAN subroutine, we based the remainder of our analysis on these two profiles. Results of the PEARSON CORR subprogram analyses revealed that for profile B, only the release parameter groupings of altitude-lead, flight path angle-lead, and heading-offset were statistically, significantly correlated. For profile D, only the groupings heading-offset and altitude-flight path angle were statistically, significantly correlated. All the remaining pairs of release parameters for both profiles were concluded to be statistically uncorrelated at the 95% confidence level and were

assigned a value of zero in the variance-covariance matrices (see Appendix G).

The correlation coefficient matrix for each of the profiles, other than B and D, showed anywhere from four to nine groupings of parameters to be significantly correlated. Since four of these six profiles were at the higher release altitude, we considered this to be a significant indication that pilots' performance at higher release altitudes were more correlated than for lower altitudes.

Research Hypothesis

Finally, we compared the results of model runs made with independent versus correlated release parameters. For both profiles, we found that the variance of bomb impacts generated from independent release parameters was greater than the variance of bomb impacts generated from correlated release parameters. Thus, for profiles B and D, we rejected our null research hypothesis and accepted the alternate hypothesis. We concluded that the variance of computer generated bomb impact distributions resulting from variation of correlated release parameters was indeed less than the variance of those distributions resulting from independent release parameters.

OBSERVATIONS

While no definite conclusions can be drawn from our study concerning specific pilot controlled release parameters

which could improve weapon delivery effectiveness, we believe that several observations and comments are warranted. The fact that release parameters are correlated could probably have been discerned merely from a casual conversation with any experienced bomber pilot. Nevertheless, we still believe that even a partial recognition of specific combinations of correlated release parameters which might improve bombing accuracy could significantly reduce sorties and aircraft exposure to enemy fire. To this end, we offer the following observations.

In our limited data source, we noted that the parameter pairs, heading-offset and flight path angle-lead, were correlated for both of the profiles investigated.

Variation of Correlated Parameters

To determine which set of correlated parameters had the most effect on accuracy, each correlated pair of release parameters was subsequently uncorrelated (covariance arbitrarily set equal to zero) for analysis. Of the two pairings which affected range error, the uncorrelation of flight path angle-lead had the most drastic effect on bombing accuracy as opposed to altitude-lead (See Appendix K). We concluded that achieving the nominal flight path angle was more important than achieving the nominal altitude at the perfect bomb release point.

Release Altitude

In reviewing the histograms of release altitude, we noted that the most frequently occurring release altitude (mode) was approximately 200 feet below the nominal values for both the 1800 and 2500 feet release altitudes. For the 2500 feet altitude, the mean of the distribution was approximately equal to the mode. However, for the 1800 feet altitude (profile A-D), the mean was some 60 feet below the mode. This again indicates that the pilots were not duplicating their release altitude performance at the 1800 feet altitude as well as they did at the 2500 feet altitude.

Hypothesis Runs

An examination of the statistical results for the hypothesis runs (profiles B and D), contained in Appendix K, provided some interesting observations.

The independent release parameter runs for profile B and for profile D were both found not to be bivariate normally distributed. This is in agreement with the conclusion made by Downs and Forseth that, in general, the distribution of computer generated impacts resulting from independent release parameters was not bivariate normal.

Examination of the profile B correlated release parameter results showed that all but one of the impact distributions were bivariate normally distributed. Profile B, with altitude-lead uncorrelated, was determined not to be bivariate normally distributed because the X and Y impact coordinates

were not independent. The bivariate normal distribution of computer generated impacts from correlated release parameters is in complete agreement with the situation found to exist for the A-10 bomb impacts recorded during the bombing accuracy evaluation tests.

The results for profile D correlated release parameters showed that none of the impact distributions were bivariate normally distributed. A possible explanation for this inconsistency is that there was one less correlated pair of parameters for the profile D variance-covariance matrices than for the profile B matrices. The profile D impacts were generated from release parameters that were almost totally independent (two out of six and one out of six parameters uncorrelated). This would then put the results from the profile D analyses in the same category as the independently generated profiles B and D. The impacts would then be expected not to be bivariate normally distributed.

RECOMMENDATIONS

As the result of our research, we offer the following suggestions for further study:

1. Test the existing Downs and Forseth model with the Brown and Callen modification with additional real world data. This would afford further insight into the arena of independent versus correlated release parameters.

2. Perform indepth analysis of the FORTRAN subroutine we incorporated into the Downs and Forseth model to confirm our reasoning as to why the other profile matrices would not function in the subroutine.

3. Perform a sensitivity analysis of the model in its present state to determine which parameters are affected most by other individual or combinations of release parameters.

4. Test the model in its present state using real world bombing data for aircraft other than the A-10.

5. Perform a systematic study of the effects of various combinations of correlation coefficients using the existing variance-covariance matrices found to be dependent (profiles other than B and D).

6. Incorporate additional subroutines to generate correlated release parameters from distributions other than multivariate normal.

APPENDIX A

A-10 BOMBING ACCURACY

EVALUATION DATA

TABLE A.1

A-10 TEST DATA--PROFILE 2A

BOMB SCORE(FEET) X +LONG -SHORT	Y +LEFT -RIGHT	AIRCRAFT SPACE POSITION XA FEET	YA FEET	ZA FEET	AIRCRAFT VELOCITY VX FT/SEC	VY FT/SEC	AIRSPEED V CAS FT/SEC	FLIGHTPATH ANGLE DEG
-17	6	-1904	-217	1617	112	46.0	256.9	-30.5
-28	58	-2031	-193	1848	392	44.0	256.9	-32.3
34	48	-1894	-178	1693	408	48.0	266.2	-31.5
-94	-9	-2101	-250	1880	672	45.0	254.8	-32.3
97	-4	-1673	-114	1765	248	22.0	265.5	-35.8
-11	31	-1862	-150	1678	104	35.0	256.1	-32.1
-56	-57	-2110	-305	1865	968	43.0	258.2	-32.1
102	-10	-1662	-180	1677	104	41.0	264.7	-34.3
30	25	-1866	-108	1740	112	30.0	262.8	-33.1

TABLE A.2

A-10 TEST DATA--PROFILE 2B

BOMB SCORE(FEET)		AIRCRAFT SPACE POSITION			AIRCRAFT VELOCITY		AIRSPEED	FLIGHTPATH
X +LONG -SHORT	Y +LEFT -RIGHT	XA FEET	YA FEET	ZA FEET	VX FT/SEC	VY FT/SEC	V CAS FT/SEC	ANGLE DEG
24	22	-2080	-167	1713	595	37.9	281.9	-29.6
13	48	-2066	-163	1720	237	39.7	281.6	-31.0
26	49	-2123	-170	1729	234	44.0	273.3	-29.6
-42	14	-2311	-200	1871	312	40.0	282.3	-30.1
21	11	-2216	-225	1796	019	45.4	282.0	-30.3
-12	50	-2241	-253	1813	806	54.9	278.9	-29.8
36	36	-2097	-264	1756	014	56.6	276.0	-30.6
1	-14	-2076	-165	1810	149	35.7	275.8	-32.0
-19	27	-2227	-224	1917	227	47.4	279.3	-31.0

TABLE A.3

A-10 TEST DATA--PROFILE 2C

BOMB SCORE(FEET) X +LONG -SHORT	BOMB SCORE(FEET) Y +LEFT -RIGHT	AIRCRAFT SPACE POSITION XA FEET	AIRCRAFT SPACE POSITION YA FEET	AIRCRAFT SPACE POSITION ZA FEET	AIRCRAFT VELOCITY VX FT/SEC	AIRCRAFT VELOCITY VY FT/SEC	AIRSPEED V CAS FT/SEC	FLIGHTPATH ANGLE DEG
140	20	-1864	-191	1672	770	72.4	295.9	-34.8
5.5	36	-1748	-238	1663	402	77.7	295.6	-37.5
-27	8	-1822	-383	1627	902	107.7	294.7	-33.5
-109	-39	-2191	-420	1886	534	87.9	287.6	-33.1
48	-10	-1681	-253	1628	477	78.6	299.8	-36.1
19	-25	-1898	-313	1673	197	79.5	293.5	-33.3
36	18	-1757	-289	1601	917	89.8	302.0	-34.6
68	-4	-1639	-325	1529	339	99.0	299.1	-34.6
10	-19	-1878	-312	1713	762	83.1	295.6	-34.7

TABLE A.4

A-10 TEST DATA--PROFILE 2D

BOMB SCORE(FEET) X +LONG -SHORT	Y +LEFT -RIGHT	AIRCRAFT SPACE POSITION XA FEET	YA FEET	$\bar{Z}$A FEET	AIRCRAFT VELOCITY VX FT/SEC	VY FT/SEC	AIRSPEED V CAS FT/SEC	FLIGHTPATH ANGLE DEG
35	61	-1839	-109	1586	851	57.2	306.4	-33.3
25	-2	-1731	-195	1504	136	63.6	309.6	-34.0
-38	-40	-1900	-278	1550	152	109.5	308.0	-32.2
22	-28	-1769	-336	1479	200	100.0	308.8	-31.7
38	-12	-1739	-261	1562	347	81.0	312.3	-34.0
-5	-19	-1717	-184	1478	773	51.8	299.3	-34.3
0	0	-1822	-199	1468	149	68.8	309.7	-32.0
-48	-14	-1993	-198	1724	499	62.2	313.9	-35.2
-6	29	-1709	-182	1459	643	67.1	310.3	-33.6

TABLE A.5

A-10 TEST DATA--PROFILE 2E

BOMB SCORE(FEET) X +LONG -SHORT	BOMB SCORE(FEET) Y +LEFT -RIGHT	AIRCRAFT SPACE POSITION XA FEET	AIRCRAFT SPACE POSITION YA FEET	AIRCRAFT SPACE POSITION ZA FEET	AIRCRAFT VELOCITY VX FT/SEC	AIRCRAFT VELOCITY VY FT/SEC	AIRSPEED V CAS FT/SEC	FLIGHTPATH ANGLE DEG
-73	-4	-1995	-81	2349	224	15.8	285.3	-41.4
-109	-5	-2292	3	2465	894	-2.5	284.6	-38.8
-10	-4	-1918	51	2489	789	-5.8	282.9	-42.5
-33	-38	-1934	-90	2212	154	10.0	274.7	-39.2
6	-23	-2049	-42	2456	950	5.9	283.6	-40.1
-43	-8	-2085	-74	2647	426	8.4	277.1	-41.6
-30	-48	-2191	-66	2668	664	2.0	285.9	-40.2
-50	-48	-2034	-159	2487	365	26.5	282.6	-41.9
-52	14	-2106	-50	2664	142	12.0	278.5	-41.7

TABLE A.6

A-10 TEST DATA--PROFILE 2F

BOMB SCORE(FEET)		AIRCRAFT SPACE POSITION			AIRCRAFT VELOCITY		AIRSPEED	FLIGHTPATH
X	Y	XA	YA	$\bar{Z}A$	VX	VY	V	ANGLE
+LONG	+LEFT						CAS	
-SHORT	-RIGHT	FEET	FEET	FEET	FT/SEC	FT/SEC	FT/SEC	DEG
34	-9	-1802	-279	2327	904	55.1	295.9	-43.6
-75	18	-2261	-161	2546	475	29.7	289.0	-40.8
58	2	-1881	-381	2361	475	81.8	303.5	-42.4
-27	-12	-2196	-166	2495	424	29.3	300.0	-41.2
47	59	-2299	-280	2510	274	59.5	291.6	-39.0
33	26	-1987	-218	2401	698	46.3	301.3	-42.8
43	2	-1931	-206	2847	830	39.4	295.3	-42.3
7	-6	-2012	-252	2483	400	49.6	298.7	-43.2
9	2	-1954	-255	2297	693	51.1	297.7	-42.2

TABLE A.7

A-10 TEST DATA--PROFILE 2G

BOMB SCORE(FEET)		AIRCRAFT SPACE POSITION			AIRCRAFT VELOCITY		AIRSPEED	FLIGHTPATH
X +LONG -SHORT	Y +LEFT -RIGHT	XA FEET	YA FEET	ZA FEET	VX FT/SEC	VY FT/SEC	V CAS FT/SEC	ANGLE DEG
-49	65	-1823	-200	2300	544	67.0	301.0	-47.1
-13	46	-1898	-127	2372	122	51.7	307.1	-44.3
-13	-39	-1918	-221	2311	562	53.5	306.7	-44.3
-11	-12	-1878	-209	2344	619	65.3	302.2	-45.2
29	28	-1793	-227	2346	408	77.9	312.3	-46.4
-5	-6	-1849	-220	2302	840	62.0	307.8	-45.1
41	-1	-1696	-297	2288	898	81.0	313.1	-47.0
-34	42	-1848	-197	2461	379	66.7	301.6	-48.3
-29	18	-1706	-274	2204	547	77.5	310.7	-46.3

TABLE A.8

A-10 TEST DATA--PROFILE 2H

BOMB SCORE(FEET)		AIRCRAFT SPACE POSITION			AIRCRAFT VELOCITY		AIRSPEED	FLIGHTPATH
X	Y	XA	YA	ZA	VX	VY	V	ANGLE
+LONG	+LEFT						CAS	
-SHORT	-RIGHT	FEET	FEET	FEET	FT/SEC	FT/SEC	FT/SEC	DEG
-49	-8	-1866	-273	2268	288	48.7	322.3	-45.2
-129	18	-2099	-323	2345	237	95.4	318.0	-43.9
5	19	-1723	-183	2308	858	81.4	325.0	-48.3
55	-3	-1747	-182	2248	034	62.2	341.0	-46.9
12	-38	-1658	-273	2215	344	80.6	322.9	-48.6
-35	2	-1770	-234	2300	656	78.0	311.4	-50.2
50	4	-1600	-259	2182	278	81.2	334.2	-47.5
0	0	-1733	-97	2300	856	78.0	321.0	-48.3
-11	22	-1724	-296	2476	320	96.6	318.0	-50.4

TABLE A.9

A-10 TEST DATA--PROFILE 4A

BOMB SCORE(FEET)		AIRCRAFT SPACE POSITION			AIRCRAFT VELOCITY		AIRSPEED	FLIGHTPATH
X	Y	XA	YA	ZA	VX	VY	V	ANGLE
+LONG	+LEFT						CAS	
-SHORT	-RIGHT	FEET	FEET	FEET	FT/SEC	FT/SEC	FT/SEC	DEG
37	-37	-1925	-105	1733	421	2.8	263.8	-31.7
29	10	-2015	-17	1725	635	17.6	260.9	-31.1
68	-48	-1938	-37	1712	080	-8.2	259.7	-33.3
-70	-30	-2208	-127	1753	354	13.3	257.2	-30.4
44	6	-1921	-178	1700	149	33.1	264.5	-30.3
-10	-11	-2052	-31	1747	982	-9.2	256.0	-29.6
45	-3	-1948	-111	1693	283	11.6	263.3	-31.6
28	-23	-1956	-146	1605	069	24.6	258.9	-32.2
25	-2	-2052	-141	1782	638	26.0	261.1	-31.8

TABLE A.10

A-10 TEST DATA--PROFILE 4B

BOMB SCORE(FEET)		AIRCRAFT SPACE POSITION			AIRCRAFT VELOCITY		AIRSPEED	FLIGHTPATH
X	Y	XA	YA	ZA	VX	VY	V	ANGLE
+LONG	+LEFT						CAS	
-SHORT	-RIGHT	FEET	FEET	FEET	FT/SEC	FT/SEC	FT/SEC	DEG
81	26	-1847	-353	1584	171	83.4	277.7	-32.7
25	78	-1834	-34	1653	757	20.2	284.8	-34.8
82	100	-2158	-181	1807	056	57.0	289.8	-31.9
-36	64	-2443	-241	1767	062	57.2	282.0	-28.6
37	-7	-2104	-335	1645	397	66.5	272.8	-30.6
20	4	-2215	-217	1703	768	42.0	299.2	-30.6
33	1	-2227	-181	1725	350	33.2	287.2	-30.8
65	-9	-2139	-217	1733	843	43.5	285.8	-31.8
95	9	-2029	-240	1675	990	54.7	291.7	-32.0

TABLE A.11

A-10 TEST DATA--PROFILE 4C

BOMB SCORE(FEET) X +LONG -SHORT	Y +LEFT -RIGHT	AIRCRAFT SPACE POSITION XA FEET	YA FEET	ZA FEET	AIRCRAFT VELOCITY VX FT/SEC	VY FT/SEC	AIRSPEED V CAS FT/SEC	FLIGHTPATH ANOLE DEG
-9	-38	-2121	21	1642	539	-5.5	290.7	-29.8
-37	28	-2060	-20	1618	552	11.2	290.2	-30.7
-42	-19	-2073	-29	1506	110	17.1	297.9	-28.4
94	-29	-1876	-46	1484	040	9.2	295.3	-28.7
36	-18	-1917	4	1562	610	1.9	290.1	-31.3
142	24	-2107	25	1582	357	16.2	295.4	-30.7
10	2	-2086	-8	1602	362	6.3	307.0	-26.0
-30	-44	-2092	-48	1715	672	10.9	294.6	-31.5
-5	0	-1956	-54	1589	406	27.8	297.5	-31.0

TABLE A.12

A-10 TEST DATA--PROFILE 4D

BOMB SCORE(FEET)		AIRCRAFT SPACE POSITION			AIRCRAFT VELOCITY		AIRSPEED	FLIGHTPATH
X	Y	XA	YA	ZA	VX	VY	V	ANGLE
+LONG	+LEFT						CAS	
-SHORT	-RIGHT	FEET	FEET	FEET	FT/SEC	FT/SEC	FT/SEC	DEG
22	29	-1806	-253	1417	421	89.5	307.5	-28.8
36	19	-1826	-76	1427	557	19.3	307.8	-29.9
-17	-5	-2008	-130	1548	344	24.0	308.6	-29.9
38	49	-1967	-302	1507	795	101.9	299.5	-26.6
67	9	-1769	-64	1336	416	22.7	310.2	-28.5
-5	45	-1978	-169	1439	123	50.3	310.6	-27.6
26	-25	-1920	-98	1529	405	9.5	305.3	-30.5
2	-29	-1900	-56	1456	779	1.9	308.2	-30.6
14	12	-1941	-103	1507	646	30.0	308.5	-30.9

TABLE A.13

A-10 TEST DATA--PROFILE 4E

BOMB SCORE(FEET) X +LONG -SHORT	Y +LEFT -RIGHT	AIRCRAFT SPACE POSITION XA FEET	YA FEET	ZA FEET	AIRCRAFT VELOCITY VX FT/SEC	VY FT/SEC	AIRSPEED V CAS FT/SEC	FLIGHTPATH ANGLE DEG
67	-49	-1908	-65	2318	976	20.2	297.7	-42.6
-78	60	-1914	-169	2255	190	48.9	286.4	-43.2
-72	50	-1782	-106	2163	190	43.0	291.5	-4[illegible].1
-6	30	-1744	-300	2185	477	73.1	297.1	-43.3
-32	-2	-1902	-182	2270	322	32.5	291.6	-41.1
-42	15	-1922	-220	2355	818	45.6	287.1	-42.1
39	43	-1815	-119	2260	555	26.5	301.1	-43.1
-20	-67	-1960	-124	2528	250	24.7	285.5	-46.3
84	8	-1772	14	2344	683	-13.8	299.1	-44.2

TABLE A.14

A-10 TEST DATA--PROFILE 4F

BOMB SCORE(FEET) X +LONG -SHORT	Y +LEFT -RIGHT	AIRCRAFT SPACE POSITION XA FEET	YA FEET	ZA FEET	AIRCRAFT VELOCITY VX FT/SEC	VY FT/SEC	AIRSPEED V CAS FT/SEC	FLIGHTPATH ANGLE DEG
-85	-7	-2134	-93	2474	864	18.8	316.4	-44.1
-15	28	-1901	-240	2257	368	57.5	320.7	-44.5
36	-24	-1870	-210	2318	008	43.1	317.1	-45.3
-27	-29	-2041	-139	2295	093	22.3	317.2	-43.2
30	-12	-1881	-174	2323	150	36.2	320.2	-45.0
-62	8	-2033	-235	2315	938	47.7	314.5	-43.7
0	8	-1850	-341	2314	637	73.5	318.5	-46.0
-33	-46	-2104	-201	2457	646	32.8	310.3	-43.6
63	27	-1920	-204	2287	797	48.0	312.5	-43.5

TABLE A.15

A-10 TEST DATA--PROFILE 4G

BOMB SCORE(FEET)		AIRCRAFT SPACE POSITION			AIRCRAFT VELOCITY		AIRSPEED	FLIGHTPATH
X	Y	XA	YA	ZA	VX	VY	V	ANGLE
+LONG	+LEFT						CAS	
-SHORT	-RIGHT	FEET	FEET	FEET	FT/SEC	FT/SEC	FT/SEC	DEG
3	24	-1963	-117	2267	934	34.0	315.4	-42.8
47	22	-1848	-89	2272	288	22.9	321.4	-44.5
-22	15	-2001	-90	2329	002	24.6	324.0	-42.5
21	44	-1854	-71	2298	130	29.9	321.4	-43.8
60	10	-1786	-98	2242	997	47.5	321.0	-46.6
82	-30	-1833	-67	2309	920	11.7	314.5	-44.2
-6	-19	-1939	22	2302	141	-8.5	318.0	-44.6
52	-34	-1911	-98	2319	213	19.8	323.1	-43.5
-4	-61	-2032	13	2382	216	-7.3	318.2	-43.1

TABLE A.16

A-10 TEST DATA--PROFILE 4H

BOMB SCORE(FEET)		AIRCRAFT SPACE POSITION			AIRCRAFT VELOCITY		AIRSPEED	FLIGHTPATH
X	Y	XA	YA	ZA	VX	VY	V	ANGLE
+LONG	+LEFT						CAS	
-SHORT	-RIGHT	FEET	FEET	FEET	FT/SEC	FT/SEC	FT/SEC	DEG
-68	-10	-1964	-49	2193	795	12.2	331.8	-41.4
6	81	-1769	-35	2155	368	12.7	340.4	-43.0
37	-33	-1747	-83	2154	155	18.4	345.7	-42.9
52	-50	-1710	-175	2097	302	43.5	337.1	-43.5
62	49	-1423	-146	1843	510	71.7	341.4	-47.0
20	5	-1790	-103	2048	379	27.0	335.9	-41.8
2	56	-1702	-290	2014	288	86.7	338.8	-41.9
27	-61	-1777	29	2250	358	-8.8	333.4	-42.9
39	-25	-1673	-61	1836	621	24.7	338.6	-42.3

APPENDIX B

MODIFICATION OF COLLAPSE

APPENDIX B

MODIFICATION OF COLLAPSE

The Downs and Forseth model program, COLLAPSE, had to be modified slightly for the analysis of the bomb impacts recorded during the A-10 bombing accuracy evaluation tests. The modifications were necessary because COLLAPSE was designed to analyze 10,000 impacts as opposed to the 144 impacts in the A-10 data. Specific modifications, by line number, are as shown below:

0180		PARAMETER INUMCELL = 50 This statement sets the initial number of cells at 50 instead of 1000.
1640	0190	IF(IICELLS+1-INUMCELL) 0200,,
2580	0410	IF(IICELLS+1-INUMCELL) 0420,, These statements provide for the maximum number of iterations to equal 50 instead of 25.

Deleted lines 3310-3510. These lines were not required since output file, PLOTDATA, was not used to plot data.

APPENDIX C

INDEPENDENCE TEST OF

BOMB IMPACTS

TABLE C.1

INDEPENDENCE TEST OF BOMB IMPACTS

Number of Impacts = 144

Distribution of Impacts in Cells

32	28
36	48

The Sample Chi-Value is 1.541 with 1 Degree of Freedom.

The Critical Chi-Squared Value is 3.841.

****
If the Chi-Squared sample value is less than Chi-Squared critical, accept X and Y are independent at the 95% level.
****

APPENDIX D

NORMALCY TESTS OF

BOMB IMPACTS

TABLE D. 1

NORMALCY TEST OF X-IMPACTS

KOLMOGOROV-SMIRNOV ANALYSIS
COMPUTING THE VALUE OF CELLS GENERATED FOR THE NORMAL

SAMPLE AVE = 7.392

SAMPLE STD = 47.847
MAX ERROR = 0.048 PROB (0.05, NUM, 144) = 0.113
(IF ERROR IS LE PROB ACCEPT THE DISTRIBUTION)

CELL NO.	X	PROBABILITY CUM	PROBABILITY PER CELL	FREQUENCY ACTUAL	FREQUENCY THEORY	ABSOLUTE ERROR	CHI SQUARE
1	-112.1	0.017	0.017	3	2.4	0.004	
2	-78.1	0.082	0.065	9	9.3	0.002	0.0120
3	-44.2	0.241	0.159	22	22.9	0.005	0.0377
4	-10.3	0.491	0.250	37	36.0	0.002	0.0281
5	23.6	0.743	0.253	43	36.4	0.048	1.2100
6	57.5	0.908	0.164	20	23.7	0.023	0.5637
7	91.4	0.976	0.068	8	9.8	0.010	0.3423
8	125.3	0.991	0.015	0	2.2	0.005	

CHI SQ 7.815 SUM 2.194

IF THE SUM IS LESS THAN CHI SQ
ACCEPT THE PROBABILITY FUNCTION
AT THE 0.050 SIGNIFICANCE LEVEL FOR
THAT PORTION OF THE DISTRIBUTION
WITH PRINTED VALUES FOR CHI SQ.

XMU = 7.392 SIGMA = 47.847

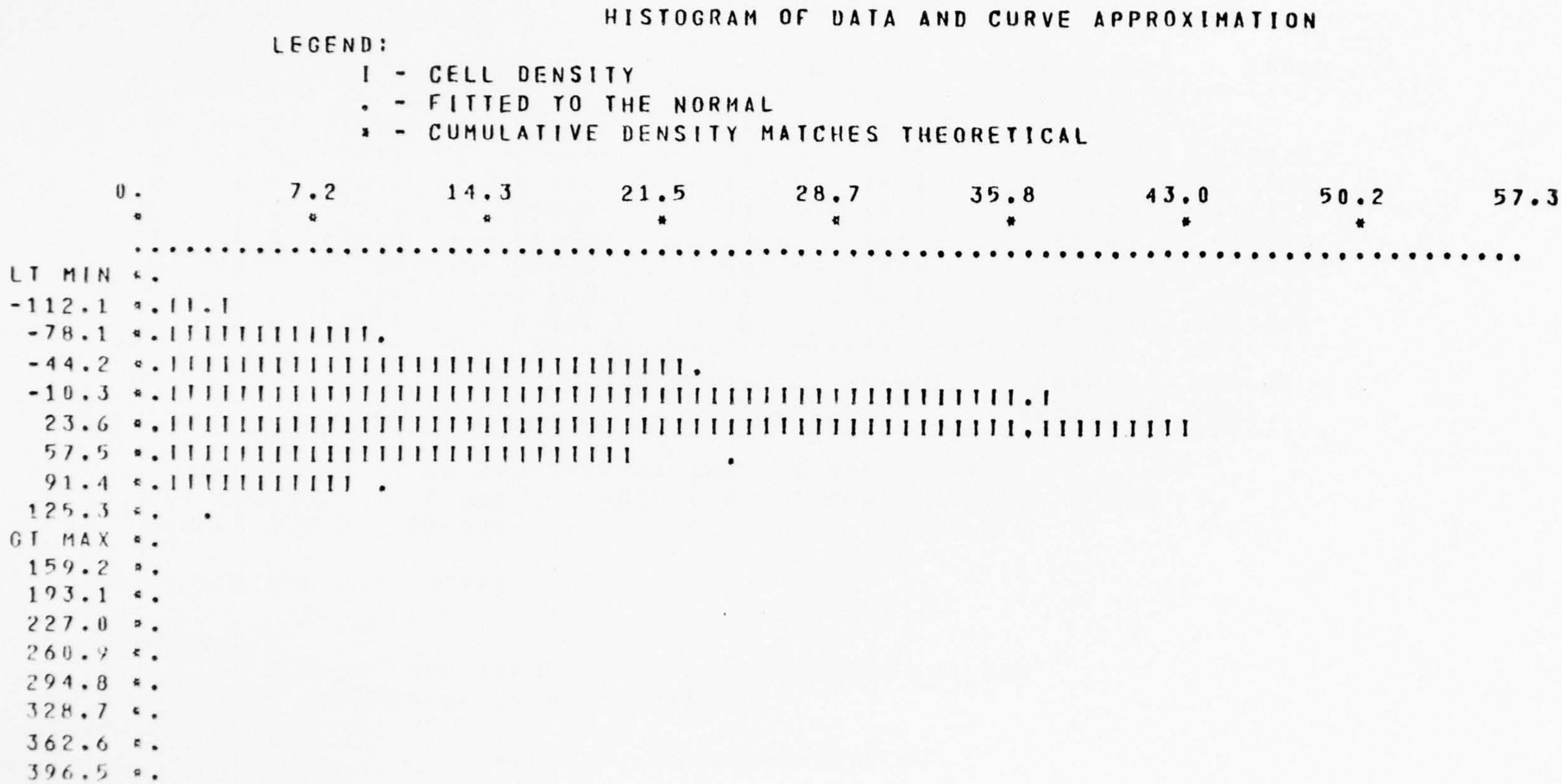


Figure D.1 Histogram of X-Impacts

TABLE D.2

NORMALCY TEST OF Y-IMPACTS

KOLMOGOROV-SMIRNOV ANALYSIS
COMPUTING THE VALUE OF CELLS GENERATED FOR THE NORMAL

SAMPLE AVE = 3.667

SAMPLE STD = 32.143
MAX ERROR = 0.032 PROB (0.05, NUM, 144) = 0.113
(IF ERROR IS LE PROB ACCEPT THE DISTRIBUTION)

CELL		PROBABILITY		FREQUENCY		ABSOLUTE	CHI
NO.	X	CUM	PER CELL	ACTUAL	THEORY	ERROR	SQUARE
1	-56.5	0.051	0.051	9	7.3	0.012	0.3962
2	-35.6	0.176	0.126	17	18.1	0.004	0.0683
3	-14.7	0.389	0.212	32	30.6	0.014	0.0650
4	6.2	0.634	0.245	38	35.3	0.032	0.2008
5	27.1	0.828	0.194	26	28.0	0.019	0.1370
6	48.0	0.933	0.105	14	15.1	0.011	0.0822
7	68.9	0.972	0.039	6	5.6	0.014	0.0358
8	89.8	0.980	0.008	0	1.2	0.006	

CHI SQ 9.488 SUM 0.985

IF THE SUM IS LESS THAN CHI SQ
ACCEPT THE PROBABILITY FUNCTION
AT THE 0.050 SIGNIFICANCE LEVEL FOR
THAT PORTION OF THE DISTRIBUTION
WITH PRINTED VALUES FOR CHI SQ.

XMU = 3.667 SIGMA = 32.143

HISTOGRAM OF DATA AND CURVE APPROXIMATION

LEGEND:

I - CELL DENSITY

. - FITTED TO THE NORMAL

* - CUMULATIVE DENSITY MATCHES THEORETICAL

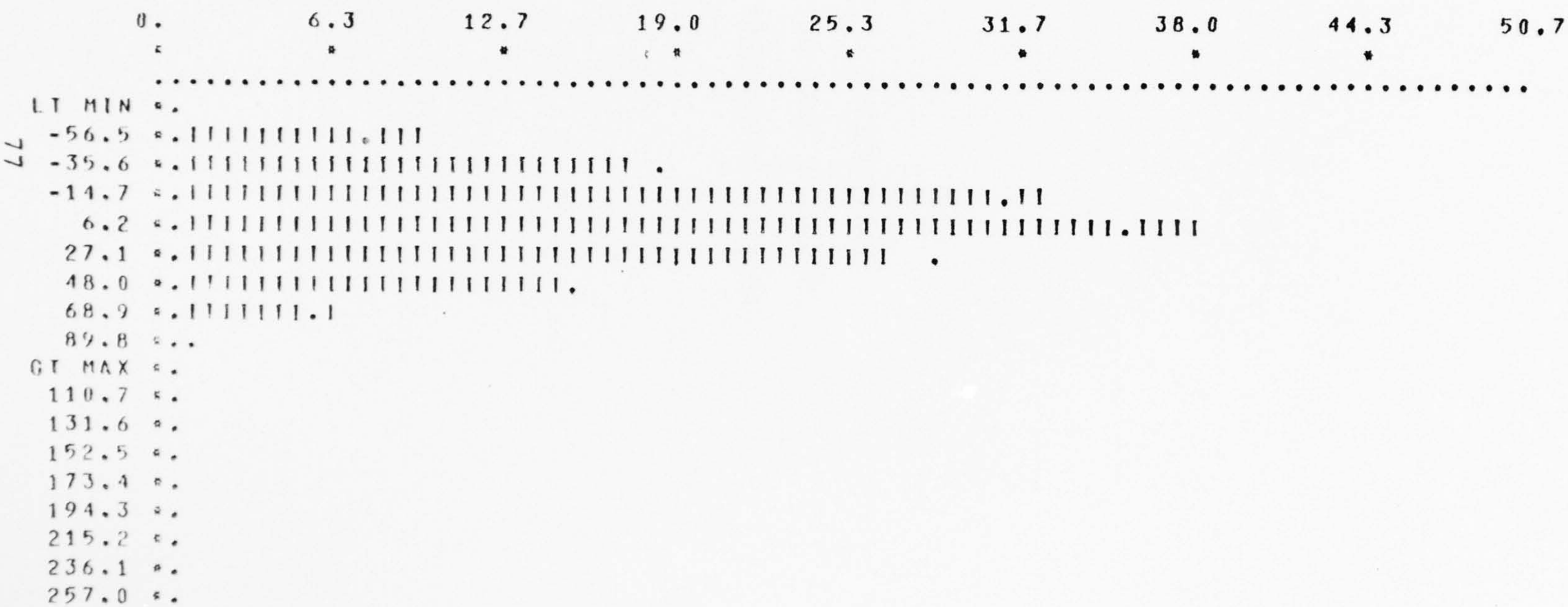


Figure D.2 Histogram of Y-Impacts

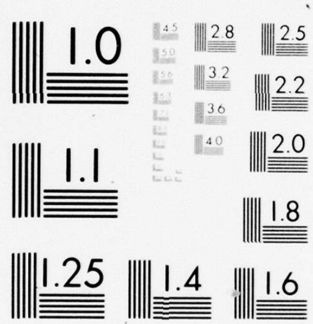
APPENDIX E

NORMALCY TEST OF

RELEASE PARAMETERS

AD-A044 185 AIR FORCE INST OF TECH WRIGHT-PATTERSON AFB OHIO SCHO--ETC F/G 19/5
THE EFFECT OF RELEASE PARAMETER CORRELATIONS ON THE DISTRIBUTIO--ETC(U)
JUN 77 H A BROWN, M H CALLEN
UNCLASSIFIED AFIT-LSSR-25-77A NL

2 OF 3
ADA044185



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

TABLE E.1

NORMALCY TEST OF LEAD/TRAIL

KOLMOGOROV-SMIRNOV ANALYSIS
COMPUTING THE VALUE OF CELLS GENERATED FOR THE NORMAL

SAMPLE AVE =-1935.238

SAMPLE STD = 165.685
MAX ERROR = 0.052 PROB (0.05, NUM, 143) = 0.113
(IF ERROR IS LE PROB ACCEPT THE DISTRIBUTION)

CELL NO.	X	PROBABILITY CUM	PER CELL	FREQUENCY ACTUAL	THEORY	ABSOLUTE ERROR	CHI SQUARE
1	-2379.2	0.014	0.014	1	1.9	0.007	
2	-2251.6	0.074	0.060	13	8.7	0.023	2.1657
3	-2124.0	0.234	0.161	20	23.1	0.002	0.4253
4	-1996.4	0.496	0.262	30	37.7	0.052	1.5621
5	-1868.8	0.759	0.263	46	37.8	0.005	1.7778
6	-1741.2	0.921	0.162	27	23.4	0.030	0.5601
7	-1613.6	0.982	0.061	6	8.8	0.011	0.9030
8	-1486.0	0.994	0.012	0	1.8	0.001	

CHI SQ 7.815 SUM 7.394

IF THE SUM IS LESS THAN CHI SQ
ACCEPT THE PROBABILITY FUNCTION
AT THE 0.050 SIGNIFICANCE LEVEL FOR
THAT PORTION OF THE DISTRIBUTION
WITH PRINTED VALUES FOR CHI SQ.

XMU =1931.681 SIGMA = 170.533

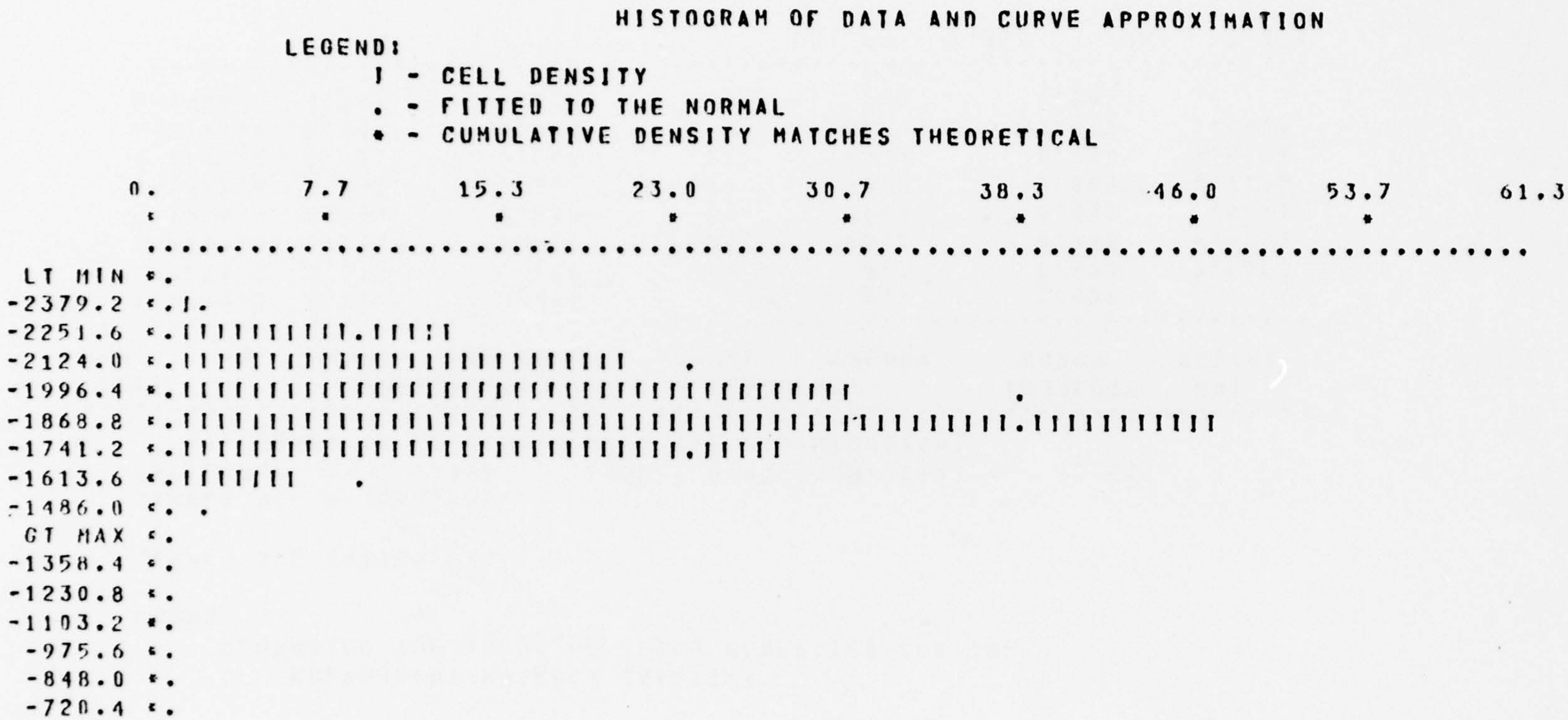


Figure E.1 Histogram of Lead/Trail

TABLE E.2

NORMALCY TEST OF OFFSET

KOLMOGOROV-SMIRNOV ANALYSIS
COMPUTING THE VALUE OF CELLS GENERATED FOR THE NORMAL

SAMPLE AVE = -166.000

SAMPLE STD = 100.971
MAX ERROR = 0.026 PROB (0.05, NUM, 144) = 0.113
(IF ERROR IS LE PROB ACCEPT THE DISTRIBUTION)

CELL NO.	X	PROBABILITY CUM	PROBABILITY PER CELL	FREQUENCY ACTUAL	FREQUENCY THEORY	ABSOLUTE ERROR	CHI SQUARE
1	-390.5	0.023	0.023	3	3.3	0.002	
2	-331.6	0.088	0.065	9	9.3	0.004	0.0122
3	-272.7	0.221	0.134	21	19.2	0.008	0.1599
4	-213.8	0.423	0.201	29	29.0	0.008	0.0000
5	-154.9	0.644	0.222	29	31.9	0.013	0.2707
6	-96.0	0.823	0.179	25	25.8	0.018	0.0237
7	-37.1	0.929	0.106	19	15.2	0.008	0.9368
8	21.8	0.963	0.034	0	4.9	0.026	
					CHI SQ 7.815	SUM	1.403

IF THE SUM IS LESS THAN CHI SQ
ACCEPT THE PROBABILITY FUNCTION
AT THE 0.050 SIGNIFICANCE LEVEL FOR
THAT PORTION OF THE DISTRIBUTION
WITH PRINTED VALUES FOR CHI SQ.

XMU =-166.000 SIGMA = 100.971

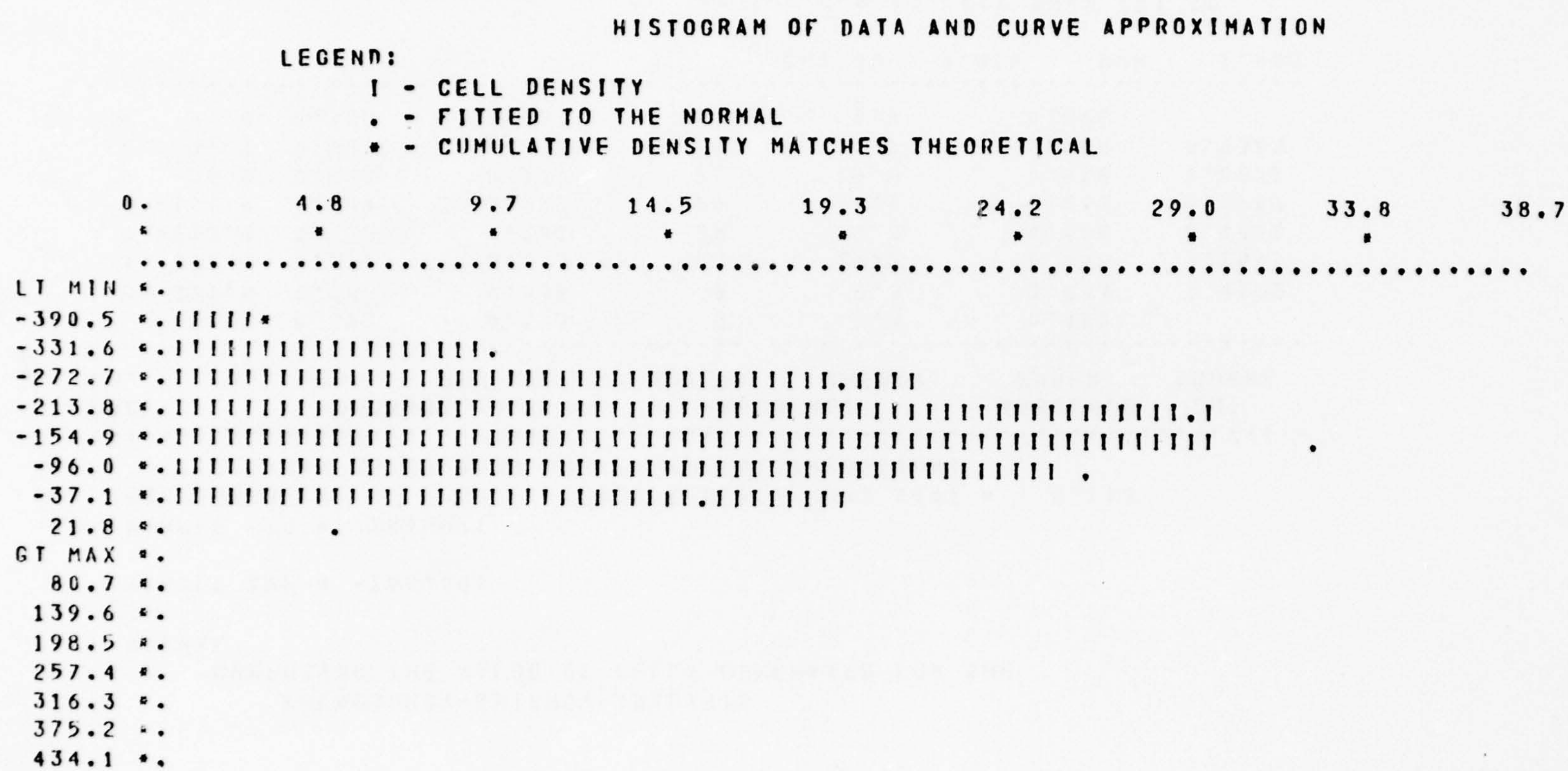


Figure E.2 Histogram of Offset

TABLE E.3

NORMALCY TEST OF AIRSPEED

KOLMOGOROV-SMIRNOV ANALYSIS

COMPUTING THE VALUE OF CELLS GENERATED FOR THE NORMAL

SAMPLE AVE = 298.101

SAMPLE STD = 21.497

MAX ERROR = 0.071 PROB (0.05, NUM, 144) = 0.113

(IF ERROR IS LE PROB ACCEPT THE DISTRIBUTION)

CELL NO.	X	PROBABILITY CUM	PROBABILITY PER CELL	FREQUENCY ACTUAL	FREQUENCY THEORY	ABSOLUTE ERROR	CHI SQUARE
1	259.9	0.041	0.041	16	6.0	0.070	16.9271
2	270.1	0.124	0.083	5	11.9	0.022	3.9948
3	280.3	0.257	0.133	18	19.2	0.014	0.0718
4	290.5	0.431	0.173	24	25.0	0.007	0.0387
5	300.7	0.613	0.183	23	26.3	0.016	0.4185
6	310.9	0.769	0.156	28	22.4	0.023	1.3896
7	321.1	0.876	0.107	19	15.4	0.047	0.8225
8	331.3	0.936	0.060	4	8.6	0.015	2.4510
9	341.5	0.955	0.019	0	2.8	0.004	
					CHI SQ 11.070	SUM	26.114

IF THE SUM IS LESS THAN CHI SQ ACCEPT THE PROBABILITY FUNCTION AT THE 0.050 SIGNIFICANCE LEVEL FOR THAT PORTION OF THE DISTRIBUTION WITH PRINTED VALUES FOR CHI SQ.

XMU = 298.101 SIGMA = 21.497

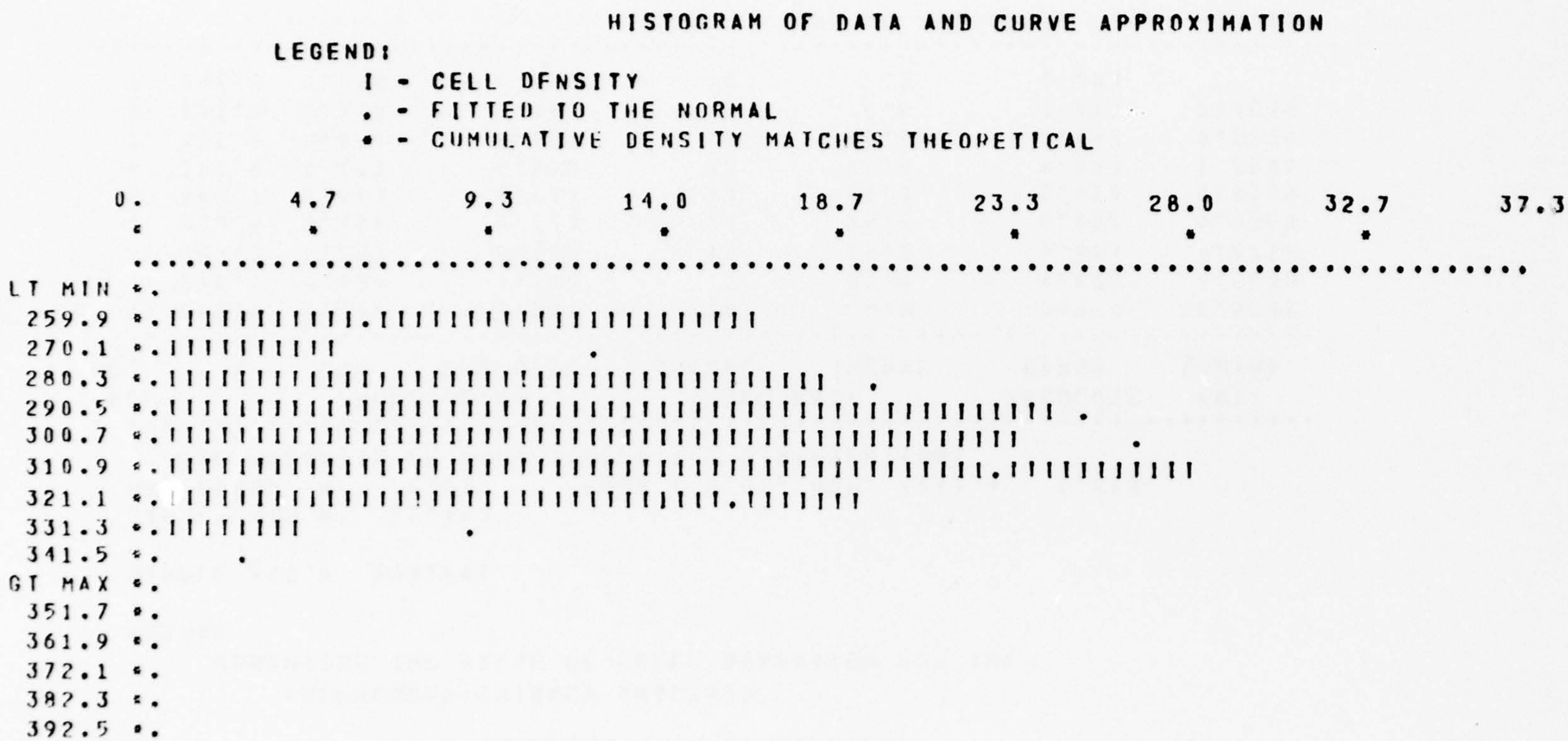


Figure E.3 Histogram of Airspeed

TABLE E.4

NORMALCY TEST OF HEADING (PROFILES A-D)

KOLMOGOROV-SMIRNOV ANALYSIS
COMPUTING THE VALUE OF CELLS GENERATED FOR THE
NORMAL

SAMPLE AVE = 5.751

SAMPLE STD = 3.731
MAX ERROR = 0.070 PROB (0.05, NUM, 72) = 0.160
(IF ERROR IS LE PROB ACCEPT THE DISTRIBUTION)

CELL NO.	X	PROBABILITY CUM	PROBABILITY PER CELL	FREQUENCY ACTUAL	FREQUENCY THEORY	ABSOLUTE ERROR	CHI SQUARE
1	-0.3	0.067	0.067	6	4.8	0.016	
2	1.9	0.203	0.136	13	9.8	0.061	1.0561
3	4.1	0.405	0.202	11	14.5	0.012	0.8512
4	6.2	0.625	0.220	20	15.8	0.070	1.0988
5	8.4	0.801	0.176	8	12.7	0.005	1.7378
6	10.5	0.905	0.104	9	7.5	0.026	0.3075
7	12.7	0.938	0.033	0	2.4	0.008	

CHI SQ 5.991 SUM 5.051

IF THE SUM IS LESS THAN CHI SQ
ACCEPT THE PROBABILITY FUNCTION
AT THE 0.050 SIGNIFICANCE LEVEL FOR
THAT PORTION OF THE DISTRIBUTION
WITH PRINTED VALUES FOR CHI SQ.

XMU = 5.751 SIGMA = 3.731

```
                         HISTOGRAM OF DATA AND CURVE APPROXIMATION
          LEGEND:
               I - CELL DENSITY
               . - FITTED TO THE NORMAL
               * - CUMULATIVE DENSITY MATCHES THEORETICAL

         0.       3.3       6.7      10.0      13.3      16.7      20.0      23.3      26.7
          *         *         *         *         *         *         *         *
          .................................................................................
 LT MIN   *.
   -0.3   *.IIIIIIIIIIIII.IIII
    1.9   *.IIIIIIIIIIIIIIIIIIIIIIIIIIII.IIIIIIIIII
    4.1   *.IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII         .
    6.2   *.IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII.IIIIIIIIIIIII
    8.4   *.IIIIIIIIIIIIIIIIIIIIIIII             .
   10.5   *.IIIIIIIIIIIIIIIIIIIIII.IIII
   12.7   *.      .
 GT MAX   *.
   14.9   *.
   17.0   *.
   19.2   *.
   21.3   *.
   23.5   *.
   25.7   *.
   27.8   *.
   30.0   *.
```

Figure E.4 Histogram of Heading (Profiles A-D)

TABLE E.5

NORMALCY TEST OF HEADING (PROFILES E-H)

KOLMOGOROV-SMIRNOV ANALYSIS
COMPUTING THE VALUE OF CELLS GENERATED FOR THE NORMAL

SAMPLE AVE = 5.864

SAMPLE STD = 4.017
MAX ERROR = 0.081 PROB (0.05, NUM, 72) = 0.160
(IF ERROR IS LE PROB ACCEPT THE DISTRIBUTION)

CELL NO.	X	PROBABILITY CUM	PROBABILITY PER CELL	FREQUENCY ACTUAL	FREQUENCY THEORY	ABSOLUTE ERROR	CHI SQUARE
1	-0.9	0.061	0.061	6	4.4	0.022	
2	1.5	0.191	0.130	11	9.3	0.045	0.2922
3	3.8	0.391	0.200	17	14.4	0.081	0.4794
4	6.2	0.615	0.224	12	16.1	0.024	1.0505
5	8.6	0.798	0.183	9	13.2	0.034	1.3235
6	10.9	0.907	0.109	15	7.8	0.066	6.5135
7	13.3	0.942	0.035	0	2.5	0.030	

CHI SQ 5.991 SUM 9.659

IF THE SUM IS LESS THAN CHI SQ
ACCEPT THE PROBABILITY FUNCTION
AT THE 0.050 SIGNIFICANCE LEVEL FOR
THAT PORTION OF THE DISTRIBUTION
WITH PRINTED VALUES FOR CHI SQ.

XMU = 5.864 SIGMA = 4.017

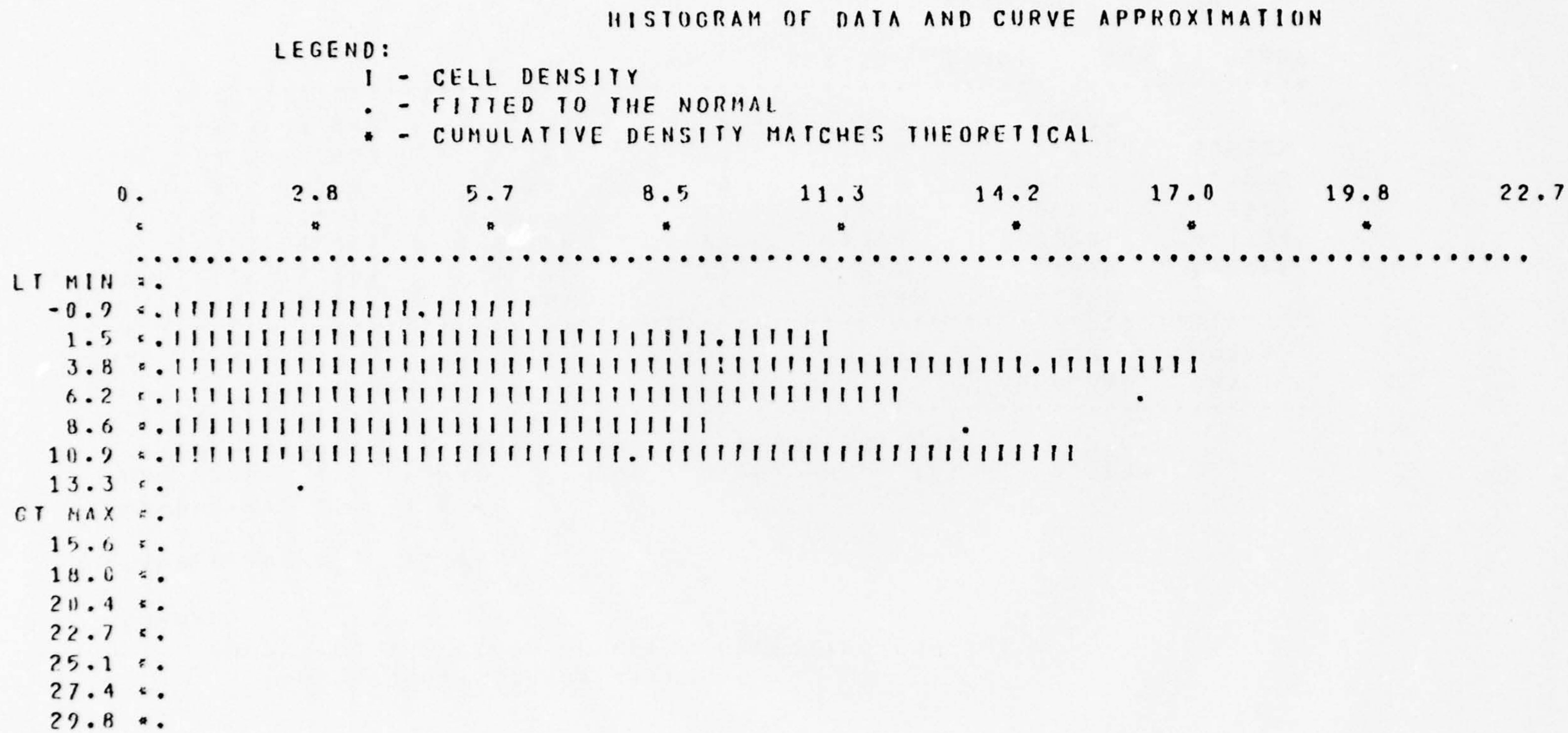


Figure E.5 Histogram of Heading (Profiles E-H)

TABLE E.6

NORMALCY TEST OF ALTITUDE (PROFILES A-D)

KOLMOGOROV-SMIRNOV ANALYSIS
COMPUTING THE VALUE OF CELLS GENERATED FOR THE
NORMAL

SAMPLE AVE = 1652.472

SAMPLE STD = 128.938
MAX ERROR = 0.049 PROB (0.05, NUM, 72) = 0.160
(IF ERROR IS LE PROB ACCEPT THE DISTRIBUTION)

CELL NO.	X	PROBABILITY CUM	PROBABILITY PER CELL	FREQUENCY ACTUAL	FREQUENCY THEORY	ABSOLUTE ERROR	CHI SQUARE
1	1384.4	0.043	0.043	3	3.1	0.001	
2	1481.3	0.173	0.131	13	9.4	0.049	1.3822
3	1578.2	0.415	0.242	12	17.4	0.026	1.6991
4	1675.1	0.691	0.276	21	19.8	0.010	0.0666
5	1772.0	0.884	0.193	17	13.9	0.033	0.6977
6	1868.9	0.948	0.064	0	4.6	0.031	

CHI SQ 3.841 SUM 3.846

IF THE SUM IS LESS THAN CHI SQ
ACCEPT THE PROBABILITY FUNCTION
AT THE 0.050 SIGNIFICANCE LEVEL FOR
THAT PORTION OF THE DISTRIBUTION
WITH PRINTED VALUES FOR CHI SQ.

XMU =1652.472 SIGMA = 128.938

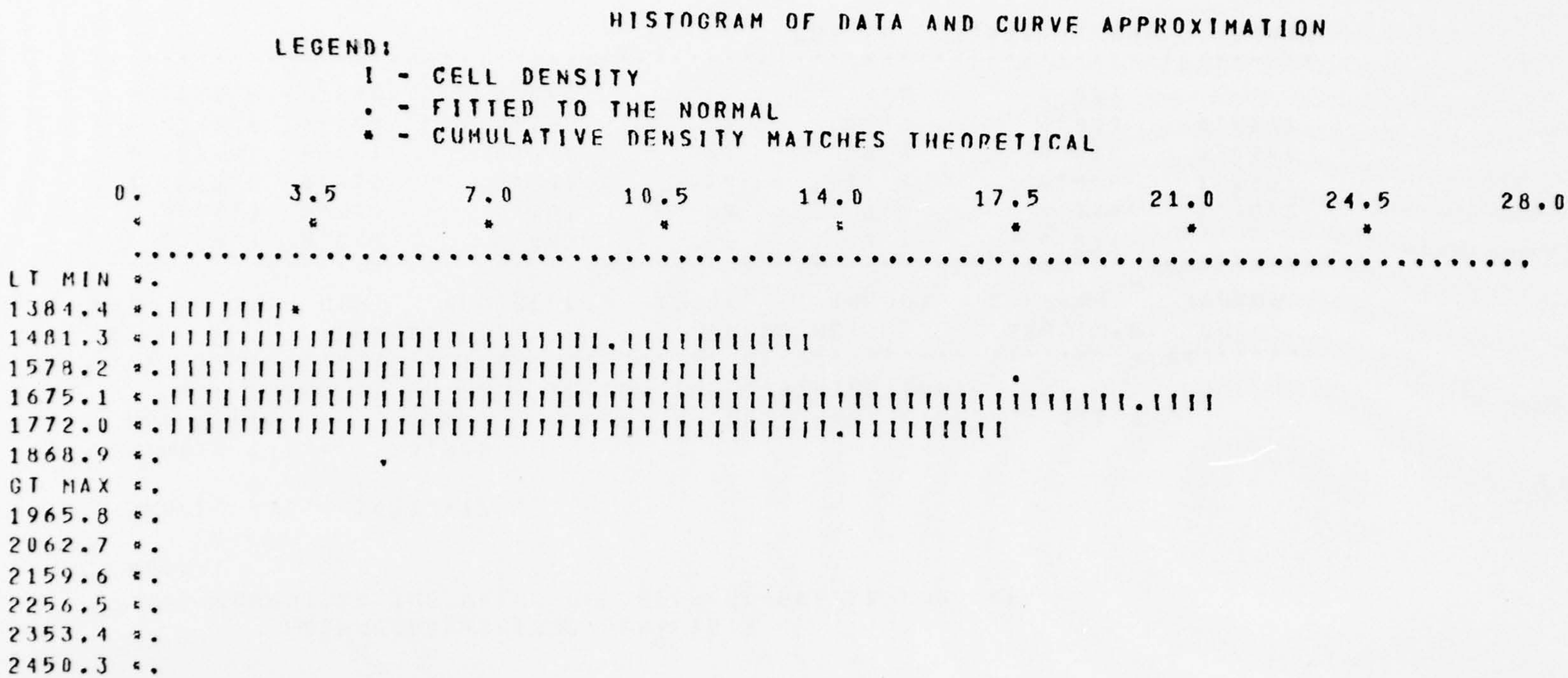


Figure E.6 Histogram of Altitude (Profiles A-D)

TABLE E.7

NORMALCY TEST OF ALTITUDE (PROFILES E-H)

KOLMOGOROV-SMIRNOV ANALYSIS
COMPUTING THE VALUE OF CELLS GENERATED FOR THE NORMAL

SAMPLE AVE = 2323.639

SAMPLE STD = 163.814
MAX ERROR = 0.088 PROB (0.05, NUM, 72) = 0.160
(IF ERROR IS LE PROB ACCEPT THE DISTRIBUTION)

CELL NO.	X	PROBABILITY CUM	PROBABILITY PER CELL	FREQUENCY ACTUAL	FREQUENCY THEORY	ABSOLUTE ERROR	CHI SQUARE
1	1920.3	0.033	0.033	2	2.4	0.005	
2	2088.9	0.199	0.165	6	11.9	0.088	2.9359
3	2257.5	0.537	0.339	37	24.4	0.088	6.5239
4	2426.1	0.849	0.311	21	22.4	0.068	0.0885
5	2594.7	0.975	0.127	5	9.1	0.011	1.8634
6	2763.3	0.995	0.020	0	1.4	0.009	
				CHI SQ	3.841	SUM	11.412

IF THE SUM IS LESS THAN CHI SQ
ACCEPT THE PROBABILITY FUNCTION
AT THE 0.050 SIGNIFICANCE LEVEL FOR
THAT PORTION OF THE DISTRIBUTION
WITH PRINTED VALUES FOR CHI SQ.

XMU =2323.639 SIGMA = 163.814

```
                         HISTOGRAM OF DATA AND CURVE APPROXIMATION
              LEGEND:
                    I - CELL DENSITY
                    . - FITTED TO THE NORMAL
                    * - CUMULATIVE DENSITY MATCHES THEORETICAL

          0.          6.2         12.3        18.5        24.7        30.8        37.0        43.2        49.3
           *           *           *           *           *           *           *           *
           ................................................................................................
 LT MIN   *.
 1920.3   *.II*
 2088.9   *.IIIIIIIIII           .
 2257.5   *.IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII.IIIIIIIIIIIIIIIIIIIIIIIII
 2426.1   *.IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII .
 2594.7   *.IIIIIIII      .
 2763.3   *.  .
 GT MAX   *.
 2931.9   *.
 3100.5   *.
 3269.1   *.
 3437.7   *.
 3606.3   *.
```

Figure E.7 Histogram of Altitude (Profiles E-H)

TABLE E.8

NORMALCY TEST OF FLIGHT PATH ANGLE (PROFILES A-D)

KOLMOGOROV-SMIRNOV ANALYSIS
COMPUTING THE VALUE OF CELLS GENERATED FOR THE
NORMAL

SAMPLE AVE = -31.635

SAMPLE STD = 2.201
MAX ERROR = 0.045 PROB (0.05, NUM, 72) = 0.160
(IF ERROR IS LE PROB ACCEPT THE DISTRIBUTION)

CELL NO.	X	PROBABILITY CUM	PROBABILITY PER CELL	FREQUENCY ACTUAL	FREQUENCY THEORY	ABSOLUTE ERROR	CHI SQUARE
1	-36.5	0.040	0.040	3	2.9	0.002	
2	-34.6	0.189	0.149	10	10.7	0.008	0.0480
3	-32.7	0.476	0.288	20	20.7	0.018	0.0248
4	-30.8	0.774	0.298	26	21.4	0.045	0.9743
5	-28.9	0.939	0.165	10	11.9	0.019	0.2998
6	-26.9	0.981	0.041	0	3.0	0.022	

CHI SQ 3.841 SUM 1.347

IF THE SUM IS LESS THAN CHI SQ
ACCEPT THE PROBABILITY FUNCTION
AT THE 0.050 SIGNIFICANCE LEVEL FOR
THAT PORTION OF THE DISTRIBUTION
WITH PRINTED VALUES FOR CHI SQ.

XMU = -31.635 SIGMA = 2.201

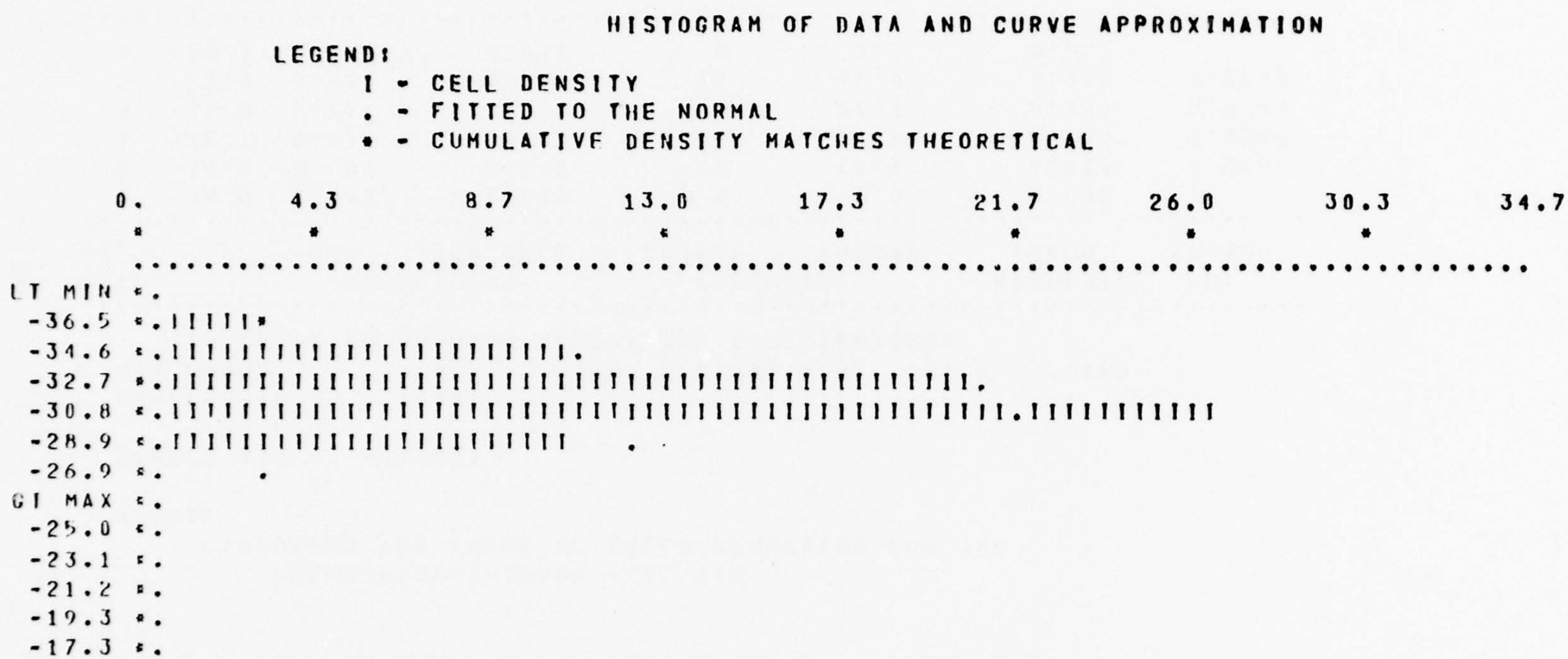


Figure E.8 Histogram of Flight Path Angle (Profiles A-D)

TABLE E.9

NORMALCY TEST OF FLIGHT PATH ANGLE (PROFILES E-H)

KOLMOGOROV-SMIRNOV ANALYSIS
COMPUTING THE VALUE OF CELLS GENERATED FOR THE
NORMAL

SAMPLE AVE = -43.881

SAMPLE STD = 2.492
MAX ERROR = 0.083 PROB (0.05, NUM, 72) = 0.160
(IF ERROR IS LE PROB ACCEPT THE DISTRIBUTION)

CELL NO.	X	PROBABILITY CUM	PROBABILITY PER CELL	FREQUENCY ACTUAL	FREQUENCY THEORY	ABSOLUTE ERROR	CHI SQUARE
1	-49.4	0.034	0.034	3	2.4	0.008	
2	-47.5	0.151	0.117	9	8.4	0.016	0.0362
3	-45.6	0.389	0.238	10	17.1	0.083	2.9672
4	-43.6	0.675	0.286	27	20.6	0.005	1.9697
5	-41.7	0.881	0.206	18	14.8	0.049	0.6780
6	-39.7	0.950	0.069	0	5.0	0.020	
					CHI SQ 3.841	SUM	5.651

IF THE SUM IS LESS THAN CHI SQ
ACCEPT THE PROBABILITY FUNCTION
AT THE 0.050 SIGNIFICANCE LEVEL FOR
THAT PORTION OF THE DISTRIBUTION
WITH PRINTED VALUES FOR CHI SQ.

XMU = -43.881 SIGMA = 2.492

```
                      HISTOGRAM OF DATA AND CURVE APPROXIMATION
          LEGEND:
               I - CELL DENSITY
               . - FITTED TO THE NORMAL
               * - CUMULATIVE DENSITY MATCHES THEORETICAL

       0.        4.5       9.0       13.5      18.0      22.5      27.0      31.5      36.0
        *         *         *         *         *         *         *         *
        ................................................................................
LT MIN *.
 -49.4 *.IIII.I
 -47.5 *.IIIIIIIIIIIIIIIII.I
 -45.6 *.IIIIIIIIIIIIIIIIIIIIII               .
 -43.6 *.IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII.IIIIIIIIIIIIII
 -41.7 *.IIIIIIIIIIIIIIIIIIIIIIIIIIIIIII.IIIIIII
 -39.7 *.          .
GT MAX *.
 -37.8 *.
 -35.8 *.
 -33.9 *.
 -32.0 *.
 -30.0 *.
```

Figure E.9 Histogram of Flight Path Angle (Profiles E-H)

APPENDIX F

THESIS-PEARSON CORR/STAT

PROGRAM

APPENDIX F

```
1000##S,R(SL) :,8,16;;,16
1005$:IDENT:WP1991,AFIT/SLG BROWN & CALLEN 77A
1010$:SELECT:SPSS/SPSS
1015RUN NAME;THESIS-PEARSON CORR/STAT
1020VARIABLE LIST; LD,OFF,ALT,VX,VY,CAS,FPA
1025VAR LABELS;LD,LEAD/
1030;OFF,OFFSET/
1035;ALT,ALTITUDE/
1040;VX,X-VELOCITY/
1045;VY,Y-VELOCITY/
1046;CAS,CALIBRATED AIRSPEED/
1047;FPA,FLIGHT PATH ANGLE
1048SUBFILE LIST;A2(9),B2(9),C2(9),D2(9),
1049;E2(9),F2(9),G2(9),H2(9),
1050;A4(9),B4(9),C4(9),D4(9),
1051;E4(9),F4(9),G4(9),H4(9)
1052COMPUTE;HDG=57.296*ATAN(VY/VX)
1053INPUT FORMAT;FREEFIELD
1055INPUT MEDIUM;CARD
1061RUN SUBFILES;(A2,A4) (B2,B4) (C2,C4) (D2,D4)
1062;(E2,E4) (F2,F4) (G2,G4) (H2,H4)
1065PEARSON CORR;LD,OFF,ALT,HDG,CAS,FPA
1075OPTIONS;3
1080STATISTICS;2
1085READ INPUT DATA
1090$:SELECTA:PLT2A
1091$:SELECTA:PLT2B
1092$:SELECTA:PLT2C
1093$:SELECTA:PLT2D
1094$:SELECTA:PLT2E
1095$:SELECTA:PLT2F
1096$:SELECTA:PLT2G
1097$:SELECTA:PLT2H
1098$:SELECTA:PLT4A
1099$:SELECTA:PLT4B
1100$:SELECTA:PLT4C
1101$:SELECTA:PLT4D
1102$:SELECTA:PLT4E
1103$:SELECTA:PLT4F
1104$:SELECTA:PLT4G
1105$:SELECTA:PLT4H
1110CONDESCRIPTIVE;ALL
1120STATISTICS;ALL
1260FINISH
1270$:ENDJOB
```

Figure F.1 Thesis-Pearson Corr/Stat Program

APPENDIX G

VARIANCE-COVARIANCE

MATRICES

TABLE G.1

PROFILE A

PEARSON CORRELATION COEFFICIENTS

	LD	OFF	ALT	HDG	CAS	FPA
LD	1.0000 (0) S=0.001	0.1014* (18)** S=0.689 ***	-0.5088 (18) S=0.031	0.1496 (18) S=0.553	0.6293 (18) S=0.005	-0.6620 (18) S=0.003
OFF	0.1014 (18) S=0.689	1.0000 (0) S=0.001	-0.3453 (18) S=0.161	-0.8260 (18) S=0.001	0.1750 (18) S=0.487	0.0572 (18) S=0.822
ALT	-0.5088 (18) S=0.031	-0.3453 (18) S=0.161	1.0000 (0) S=0.001	0.1517 (18) S=0.548	-0.3258 (18) S=0.187	-0.1405 (18) S=0.578
HDG	0.1496 (18) S=0.553	-0.8260 (18) S=0.001	0.1517 (18) S=0.548	1.0000 (0) S=0.001	-0.0279 (18) S=0.912	-0.1647 (18) S=0.514
CAS	0.6293 (18) S=0.005	0.1750 (18) S=0.487	-0.3258 (18) S=0.187	-0.0279 (18) S=0.912	1.0000 (0) S=0.001	-0.3801 (18) S=0.120
FPA	-0.6620 (18) S=0.003	0.0572 (18) S=0.822	-0.1405 (18) S=0.578	-0.1647 (18) S=0.514	-0.3801 (18) S=0.120	1.0000 (0) S=0.001

*Correlation coefficient
**Number of cases
***Alpha level of statistical significance

TABLE G.2

PROFILE A VARIANCE-COVARIANCE MATRIX

	ALT	CAS	FPA	LEAD	HDG	OFFSET
ALT	4852.850	0.	0.	-4923.060	0.	0.
CAS	0.	13.700	0.	323.540	0.	0.
FPA	0.	0.	2.220	-136.880	0.	0.
LEAD	-4923.060	323.540	-136.880	19289.180	0.	0.
HDG	0.	0.	0.	0.	7.434	-167.460
OFFSET	0.	0.	0.	0.	-167.460	5528.540

TABLE G.3

PROFILE B

PEARSON CORRELATION COEFFICIENTS

	LD	OFF	ALT	HDG	CAS	FPA
LD	1.0000	0.1345	-0.6647	0.0585	-0.1126	-0.7927
	(0)	(18)	(18)	(18)	(18)	(18)
	S=0.001	S=0.595	S=0.003	S=0.818	S=0.656	S=0.001
OFF	0.1345	1.0000	0.1702	-0.9040	0.2538	-0.3221
	(18)	(0)	(18)	(18)	(18)	(18)
	S=0.595	S=0.001	S=0.500	S=0.001	S=0.310	S=0.192
ALT	-0.6647	0.1702	1.0000	-0.2095	-0.0708	0.3770
	(18)	(18)	(0)	(18)	(18)	(18)
	S=0.003	S=0.500	S=0.001	S=0.404	S=0.780	S=0.123
HDG	0.0585	-0.9040	-0.2095	1.0000	-0.3411	0.1705
	(18)	(18)	(18)	(0)	(18)	(18)
	S=0.818	S=0.001	S=0.404	S=0.001	S=0.166	S=0.499
CAS	-0.1126	0.2538	-0.0708	-0.3411	1.0000	-0.1911
	(18)	(18)	(18)	(18)	(0)	(18)
	S=0.656	S=0.310	S=0.780	S=0.166	S=0.001	S=0.447
FPA	-0.7927	-0.3221	0.3770	0.1705	-0.1911	1.0000
	(18)	(18)	(18)	(18)	(18)	(0)
	S=0.001	S=0.192	S=0.123	S=0.499	S=0.447	S=0.001

TABLE G.4

PROFILE B VARIANCE-COVARIANCE MATRIX

	ALT	CAS	FPA	LEAD	HDG	OFFSET
ALT	6756.130	0.	0.	-8017.400	0.	0.
CAS	0.	44.640	0.	0.	0.	0.
FPA	0.	0.	1.980	-163.830	0.	0.
LEAD	-8017.400	0.	-163.830	21535.680	0.	0.
HDG	0.	0.	0.	0.	3.310	-115.280
OFFSET	0.	0.	0.	0.	-115.280	4913.000

TABLE G.5

PROFILE C

PEARSON CORRELATION COEFFICIENTS

	LD	OFF	ALT	HDG	CAS	FPA
LD	1.0000	-0.5042	-0.3115	0.6240	0.3653	-0.6851
	(0)	(18)	(18)	(18)	(18)	(18)
	S=0.001	S=0.033	S=0.208	S=0.006	S=0.136	S=0.002
OFF	-0.5042	1.0000	-0.4951	-0.9688	0.0250	0.7305
	(18)	(0)	(18)	(18)	(18)	(18)
	S=0.033	S=0.001	S=0.037	S=0.001	S=0.922	S=0.001
ALT	-0.3115	-0.4951	1.0000	0.3772	-0.4181	-0.3726
	(18)	(18)	(0)	(18)	(18)	(18)
	S=0.208	S=0.037	S=0.001	S=0.123	S=0.084	S=0.128
HDG	0.6240	-0.9688	0.3772	1.0000	0.0957	-0.8043
	(18)	(18)	(18)	(0)	(18)	(18)
	S=0.006	S=0.001	S=0.123	S=0.001	S=0.706	S=0.001
CAS	0.3653	0.0250	-0.4181	0.0957	1.0000	0.1058
	(18)	(18)	(18)	(18)	(0)	(18)
	S=0.136	S=0.922	S=0.084	S=0.706	S=0.001	S=0.676
FPA	-0.6851	0.7305	-0.3726	-0.8043	0.1058	1.0000
	(18)	(18)	(18)	(18)	(18)	(0)
	S=0.002	S=0.001	S=0.128	S=0.001	S=0.676	S=0.001

TABLE G.6

PROFILE C VARIANCE-COVARIANCE MATRIX

	ALT	CAS	FPA	LEAD	HDG	OFFSET
ALT	8332.471	0.	0.	0.	0.	-7051.608
CAS	0.	21.377	0.	0.	0.	0.
FPA	0.	0.	8.803	-333.296	-11.987	338.167
LEAD	0.	0.	-333.296	26886.261	514.000	12901.562
HDG	0.	0.	-11.987	514.000	25.234	-759.371
OFFSET	-7051.608	0.	338.167	12901.562	-759.371	24348.526

TABLE G.7

PROFILE D

PEARSON CORRELATION COEFFICIENTS

	LD	OFF	ALT	HDG	CAS	FPA
LD	1.0000 (0) S=0.001	-0.1493 (18) S=0.554	-0.4041 (18) S=0.096	0.1852 (18) S=0.462	-0.0096 (18) S=0.970	-0.4258 (18) S=0.078
OFF	-0.1493 (18) S=0.554	1.0000 (0) S=0.001	-0.2196 (18) S=0.381	-0.9460 (18) S=0.001	0.0913 (18) S=0.718	0.1057 (18) S=0.676
ALT	-0.4041 (18) S=0.096	-0.2196 (18) S=0.381	1.0000 (0) S=0.001	0.1947 (18) S=0.439	0.1739 (18) S=0.490	-0.5687 (18) S=0.014
HDG	0.1852 (18) S=0.462	-0.9460 (18) S=0.001	0.1947 (18) S=0.439	1.0000 (0) S=0.001	-0.1094 (18) S=0.666	-0.0976 (18) S=0.700
CAS	-0.0096 (18) S=0.970	0.0913 (18) S=0.718	0.1739 (18) S=0.490	-0.1094 (18) S=0.666	1.0000 (0) S=0.001	-0.2750 (18) S=0.269
FPA	-0.4258 (18) S=0.078	0.1057 (18) S=0.676	-0.5687 (18) S=0.014	-0.0976 (18) S=0.700	-0.2750 (18) S=0.269	1.0000 (0) S=0.001

TABLE G.8

PROFILE D VARIANCE-COVARIANCE MATRIX

	ALT	CAS	FPA	LEAD	HDG	OFFSET
ALT	6796.706	0.	-116.309	0.	0.	0.
CAS	0.	13.823	0.	0.	0.	0.
FPA	-116.309	0.	6.154	0.	0.	0.
LEAD	0.	0.	0.	10246.693	0.	0.
HDG	0.	0.	0.	0.	16.528	-325.118
OFFSET	0.	0.	0.	0.	-325.118	7146.134

TABLE G.9

PROFILE E

PEARSON CORRELATION COEFFICIENTS

	LD	OFF	ALT	HDG	CAS	FPA
LD	1.0000 (0) S=0.001	-0.3917 (18) S=0.108	-0.7513 (18) S=0.001	0.4802 (18) S=0.044	0.6280 (18) S=0.005	-0.6467 (18) S=0.004
OFF	-0.3917 (18) S=0.108	1.0000 (0) S=0.001	0.4405 (18) S=0.067	-0.9330 (18) S=0.001	-0.2299 (18) S=0.359	0.2318 (18) S=0.355
ALT	-0.7513 (18) S=0.001	0.4405 (18) S=0.067	1.0000 (0) S=0.001	-0.5415 (18) S=0.020	-0.5283 (18) S=0.024	0.1920 (18) S=0.445
HDG	0.4802 (18) S=0.044	-0.9330 (18) S=0.001	-0.5415 (18) S=0.020	1.0000 (0) S=0.001	0.2821 (18) S=0.257	-0.3504 (18) S=0.154
CAS	0.6280 (18) S=0.005	-0.2299 (18) S=0.359	-0.5283 (18) S=0.024	0.2821 (18) S=0.257	1.0000 (0) S=0.001	-0.4812 (18) S=0.043
FPA	-0.6467 (18) S=0.004	0.2318 (18) S=0.355	0.1920 (18) S=0.445	-0.3504 (18) S=0.154	-0.4812 (18) S=0.043	1.0000 (0) S=0.001

TABLE G.10

PROFILE E VARIANCE-COVARIANCE MATRIX

	ALT	CAS	FPA	LEAD	HDG	OFFSET
ALT	26535.977	-654.344	0.	17855.000	-286.176	0.
CAS	-654.344	57.805	-6.813	696.553	0.	0.
FPA	0.	-6.813	3.468	-175.691	0.	0.
LEAD	17855.000	696.553	-175.691	21283.663	227.251	0.
HDG	-286.176	0.	0.	227.251	10.524	-260.670
OFFSET	0.	0.	0.	0.	-260.670	7417.794

TABLE G. 11

PROFILE F

PEARSON CORRELATION COEFFICIENTS

	LD	OFF	ALT	HDG	CAS	FPA
LD	1.0000 (0) S=0.001	-0.4169 (18) S=0.085	-0.4180 (18) S=0.084	0.4477 (18) S=0.062	0.4649 (18) S=0.052	-0.7439 (18) S=0.001
OFF	-0.4169 (18) S=0.085	1.0000 (0) S=0.001	0.2117 (18) S=0.399	-0.9806 (18) S=0.001	0.1731 (18) S=0.492	0.0088 (18) S=0.972
ALT	-0.4180 (18) S=0.084	0.2117 (18) S=0.399	1.0000 (0) S=0.001	-0.2319 (18) S=0.354	-0.5848 (18) S=0.011	0.4919 (18) S=0.038
HDG	0.4477 (18) S=0.062	-0.9806 (18) S=0.001	-0.2319 (18) S=0.354	1.0000 (0) S=0.001	-0.1477 (18) S=0.559	-0.0207 (18) S=0.935
CAS	0.4649 (18) S=0.052	0.1731 (18) S=0.492	-0.5848 (18) S=0.011	-0.1477 (18) S=0.559	1.0000 (0) S=0.001	-0.8182 (18) S=0.001
FPA	-0.7439 (18) S=0.001	0.0088 (18) S=0.972	0.4919 (18) S=0.038	-0.0207 (18) S=0.935	-0.8182 (18) S=0.001	1.0000 (0) S=0.001

TABLE G.12

PROFILE F VARIANCE-COVARIANCE MATRIX

	ALT	CAS	FPA	LEAD	HDG	OFFSET
ALT	20364.997	-894.853	118.674	0.	0.	0.
CAS	-894.853	114.978	-14.832	0.	0.	0.
FPA	118.674	-14.832	2.858	-175.587	0.	0.
LEAD	0.	0.	-175.587	19498.487	0.	0.
HDG	0.	0.	0.	0.	5.797	-164.307
OFFSET	0.	0.	0.	0.	-164.307	4842.618

TABLE G.13

PROFILE G

PEARSON CORRELATION COEFFICIENTS

	LD	OFF	ALT	HDG	CAS	FPA
LD	1.0000	-0.6806	-0.4042	0.7049	-0.2611	-0.7578
	(0)	(18)	(18)	(18)	(18)	(18)
	S=0.001	S=0.002	S=0.096	S=0.001	S=0.295	S=0.001
OFF	-0.6806	1.0000	0.1139	-0.9514	0.6239	0.6198
	(18)	(0)	(18)	(18)	(18)	(18)
	S=0.002	S=0.001	S=0.653	S=0.001	S=0.006	S=0.006
ALT	-0.4042	0.1139	1.0000	-0.0237	-0.3466	-0.0851
	(18)	(18)	(0)	(18)	(18)	(18)
	S=0.096	S=0.653	S=0.001	S=0.926	S=0.159	S=0.737
HDG	0.7049	-0.9514	-0.0237	1.0000	-0.6642	-0.7314
	(18)	(18)	(18)	(0)	(18)	(18)
	S=0.001	S=0.001	S=0.926	S=0.001	S=0.003	S=0.001
CAS	-0.2611	0.6239	-0.3466	-0.6642	1.0000	0.5787
	(18)	(18)	(18)	(18)	(0)	(18)
	S=0.295	S=0.006	S=0.159	S=0.003	S=0.001	S=0.012
FPA	-0.7578	0.6198	-0.0851	-0.7314	0.5787	1.0000
	(18)	(18)	(18)	(18)	(18)	(0)
	S=0.001	S=0.006	S=0.737	S=0.001	S=0.012	S=0.001

TABLE G.14

PROFILE G VARIANCE-COVARIANCE MATRIX

	ALT	CAS	FPA	LEAD	HDG	OFFSET
ALT	3204.536	0.	0.	0.	0.	0.
CAS	0.	57.908	7.241	0.	-21.230	436.521
FPA	0.	7.241	2.704	-111.735	-5.051	93.702
LEAD	0.	0.	-111.735	8038.588	265.451	-5610.098
HDG	0.	-21.230	-5.051	265.451	17.641	-367.388
OFFSET	0.	436.521	93.702	-5610.098	-367.388	8452.487

TABLE G.15

PROFILE H

PEARSON CORRELATION COEFFICIENTS

	LD	OFF	ALT	HDG	CAS	FPA
LD	1.0000 (0) S=0.001	0.0611 (18) S=0.810	-0.5091 (18) S=0.031	0.1635 (18) S=0.517	0.3800 (18) S=0.120	-0.3389 (18) S=0.169
OFF	0.0611 (18) S=0.810	1.0000 (0) S=0.001	-0.3407 (18) S=0.167	-0.8601 (18) S=0.001	0.5067 (18) S=0.032	0.4879 (18) S=0.040
ALT	-0.5091 (18) S=0.031	-0.3407 (18) S=0.167	1.0000 (0) S=0.001	0.3454 (18) S=0.160	-0.6992 (18) S=0.001	-0.5082 (18) S=0.031
HDG	0.1635 (18) S=0.517	-0.8601 (18) S=0.001	0.3454 (18) S=0.160	1.0000 (0) S=0.001	-0.5545 (18) S=0.017	-0.7229 (18) S=0.001
CAS	0.3800 (18) S=0.120	0.5067 (18) S=0.032	-0.6992 (18) S=0.001	-0.5545 (18) S=0.017	1.0000 (0) S=0.001	0.6144 (18) S=0.007
FPA	-0.3389 (18) S=0.169	0.4879 (18) S=0.040	-0.5082 (18) S=0.031	-0.7229 (18) S=0.001	0.6144 (18) S=0.007	1.0000 (0) S=0.001

TABLE G.16

PROFILE H VARIANCE-COVARIANCE MATRIX

	ALT	CAS	FPA	LEAD	HDG	OFFSET
ALT	27280.144	-1157.767	-256.874	11778.582	0.	0.
CAS	-1157.767	100.498	18.851	0.	-25.948	536.396
FPA	-256.874	18.851	9.366	0.	-10.327	157.647
LEAD	11778.582	0.	0.	19620.369	0.	0.
HDG	0.	-25.948	10.327	0.	21.787	-423.900
OFFSET	0.	536.396	157.647	0.	-423.900	11148.735

APPENDIX H

INPUT DATA FILE

APPENDIX H

INPUT DATA FILE

The Downs and Forseth input file, DATAN5.5, was modified slightly to provide for the input of a variance-covariance matrix. A sample of the modified file is shown in Figure H.1. An explanation of each line entry is provided below.

Line 10 Input data file name--MVN.

CONTROL PARAMETERS

Line 20 Bomb type. Data codes: 1--MK-106, 2--BDU-33, and 3--MK-82.

Line 30 Latitude of target in decimal degrees.

Line 40 Target altitude in feet relative to MSL.

Line 50 Bomb ejection velocity in feet per second.

Line 60 Bomb release time delay in seconds.

Line 70 Number of bombs to be dropped.

PROGRAM PARAMETERS

Line 80 Random number generator seed.

Line 90 Value of T used for computing iteration time.

Line 100 Initial cell size in feet.

RELEASE PARAMETERS

Line 110 Altitude mean in feet above target followed by altitude variance-covariance string.

Line 120	True airspeed mean in feet per second followed by airspeed variance-covariance string.
Line 130	Pitch parameter (flight path angle) in degrees followed by pitch variance-covariance string.
Line 140	Lead/trail mean in feet followed by lead/trail variance-covariance string.
Line 150	Heading mean in degrees followed by heading variance-covariance string.
Line 160	Offset mean in feet followed by offset variance-covariance string.
Lines 170-190	Three lines for comments to be printed on graphics output. Fields are 19 characters in length terminated by back slash (\).

The release parameter entries with variance-covariance strings are further illustrated in Table H.1. Format is freefield.

```
010 MVN
020 2.0
030 35.0
040 2000.0
050 0.0
060 0.0
070 10000.0
080 100.0
090 4.0
100 10.0
110 3745.389 6756.13 0.0 0.0 -8017.4 0.0 0.0
120 282.383 0.0 44.64 0.0 0.0 0.0 0.0
130 -30.989 0.0 0.0 1.98 -163.83 0.0 0.0
140 8.83 -8017.4 0.0 -163.83 21535.68 0.0 0.0
150 6.197 0.0 0.0 0.0 0.0 3.31 -115.28
160 -212.778 0.0 0.0 0.0 0.0 -115.28 4913.0
170 PROFILE B\
180 CORRELATED RELEASE\
190 PARAMETERS\
```

Figure H.1 Sample Input Data File

TABLE H.1

INPUT FILE FORMAT

Parameters	Line No.	Mean for Parameter *	Variance-Covariance Matrix					
			(1) Altitude **	(2) Airspeed ***	(3) Pitch	(4) Lead/Trail	(5) Heading	(6) Offset
Altitude (1)	110	μ_1	σ_{11}^2	σ_{12}^2	σ_{13}^2	σ_{14}^2	σ_{15}^2	σ_{16}^2
Airspeed (2)	120	μ_2	σ_{21}^2	σ_{22}^2	σ_{23}^2	σ_{24}^2	σ_{25}^2	σ_{26}^2
Pitch (3)	130	μ_3	σ_{31}^2	σ_{32}^2	σ_{33}^2	σ_{34}^2	σ_{35}^2	σ_{36}^2
Lead/Trail (4)	140	μ_4	σ_{41}^2	σ_{42}^2	σ_{43}^2	σ_{44}^2	σ_{45}^2	σ_{46}^2
Heading (5)	150	μ_5	σ_{51}^2	σ_{52}^2	σ_{53}^2	σ_{54}^2	σ_{55}^2	σ_{56}^2
Offset (6)	160	μ_6	σ_{61}^2	σ_{62}^2	σ_{63}^2	σ_{64}^2	σ_{65}^2	σ_{66}^2

* Mean value for appropriate parameter, e.g., μ_1 is the altitude mean.

** Diagonal entries are values for variance of each of the respective release parameters, e.g., σ_{11}^2, is the variance for altitude.

*** Off-diagonal entries are values for covariance between parameters in intersecting rows and columns, e.g., σ_{12}^2, is the covariance between altitude and airspeed ($\sigma_{12}^2 = \sigma_{21}^2$).

APPENDIX I

TEST OF MULTIVARIATE NORMAL

FORTRAN SUBROUTINE

APPENDIX I

TEST OF MULTIVARIATE NORMAL FORTRAN SUBROUTINE

A possible error was detected in one of the FORTRAN statements of the Multivariate Normal Subroutine. This statement was:

```
6950    25 SUM1 = SUM1 + D(I+1,K).
```

Comparing this statement to the theoretical formula that was its basis (20:98) showed that one coefficient of the product sum had been omitted. Since we could not determine if this was an intentional omission with some theoretical basis or a typographical error, we modified the FORTRAN statement to agree with the theoretical formula as follows:

```
6950    25 SUM1 = SUM1 + D(I+1,K)*D(J,K).
```

The two versions of the subroutine were then tested using the profile D variance-covariance matrix. Comparison of the results are shown in Table I.2. Examination of the results showed that there was little difference in the statistics for the 100 observations which were generated by each version of the subroutine. Based on these results, it was impossible to conclude that either one of the versions was more "accurate."

We tested our modified version of the subroutine by inserting the variance-covariance matrix for profile B. One row of the correlation matrix for profile B had a sum of coefficients of determination slightly in excess of one (approximately 1.07). Theoretical aspects discussed in our conclusions indicated that a row sum in excess of one showed the assumption of independence to be invalid for a multivariate normal distribution. Since this was the case, our modified version of the subroutine did not run with the variance-covariance matrix for profile B. Based on the fact that the unmodified version of the subroutine generated the C matrix from a "slightly" dependent variance-covariance matrix, we concluded that our modified version of the subroutine was accurate.

We then incorporated the modified FORTRAN statement (Line 6950) into the subroutine in the bombing model, STHESIS. The test of the research hypothesis was again performed for profile D using our modified version of the subroutine. This was the only profile whose correlation coefficient matrix had no row sum of coefficients of determination in excess of one. As previously indicated, profile B had a row sum of coefficients of determination in excess of one and, therefore, would not run in our modified version of the subroutine. However, the covariance for each respective correlated pair of release parameters was set equal to zero (which reduced the row sum of coefficients of determination to less than one) to

investigate the effects of uncorrelating parameters previously analyzed using the unmodified version of the subroutine. Results of these analyses are shown in Appendix K. Comparing the results from the unmodified and modified versions of the subroutine showed there was only minor variation between the two. Results of the modified version runs also supported our research hypothesis that the variance of computer generated impacts resulting from correlated release parameters was less than the variance of the impacts resulting from the independent release parameters.

TABLE I.1 TEST OF MULTIVARIATE NORMAL FORTRAN SUBROUTINE

Profile B

Number of Observations = 100

Release Parameters		Mean*	Variance-Covariance Matrix					
			ALT	CAS	FPA	LEAD	HDG	OFFSET
ALT	Actual	3745.389	6756.13 **	0.0 ***	0.0	-8017.4	0.0	0.0
	Subroutine	3739.064	6168.44	-127.21	4.94	-7907.03	-2.10	-427.65
CAS	Actual	282.383		44.64	0.0	0.0	0.0	0.0
	Subroutine	282.953		50.461	-0.37	290.06	-0.39	19.37
FPA	Actual	-30.989			1.98	-163.83	0.0	0.0
	Subroutine	-30.917			1.77	-88.53	0.15	-4.48
LEAD	Actual	8.83				21535.68	0.0	0.0
	Subroutine	10.53				22444.85	-26.56	1496.14
HDG	Actual	6.197					3.31	-115.28
	Subroutine	6.401					2.98	-98.82
OFFSET	Actual	-212.778						4913.00
	Subroutine	-220.808						4318.23

* Values of actual and generated mean for each of the release parameters.

**E.g., diagonal entries are a comparison of actual and subroutine generated values for variance.

***E.g., off-diagonal entries are a comparison of actual and subroutine generated values for covariance between parameters in intersecting rows and columns.

TABLE I.2 RESULTS OF MODIFIED AND UNMODIFIED SUBROUTINES

Profile D   Number of Observations = 100

Release Parameters		Mean*	Variance-Covariance Matrix					
			ALT	CAS	FPA	LEAD	HDG	OFFSET
			**	***				
ALT	Unmodified	3492.322	6205.470	-71.03	-106.46	-120.13	-4.72	-366.78
	Modified	3492.322	6205.470	-71.03	-99.10	-120.13	-4.72	-366.78
CAS	Unmodified	308.345		15.624	2.52	8.36	-0.49	12.50
	Modified	308.345		15.624	0.92	8.36	-0.49	12.50
FPA	Unmodified	-31.066			5.525	-11.96	0.51	-0.87
	Modified	-31.097			5.299	-13.04	0.57	-2.29
LEAD	Unmodified	85.628				9958.536	-54.03	1059.20
	Modified	85.628				9958.536	-54.03	1059.20
HDG	Unmodified	7.363					14.879	-282.54
	Modified	7.363					14.879	-282.54
OFFSET	Unmodified	-187.199						6235.716
	Modified	-187.199						6235.716

* Values of mean for each release parameter generated by modified and unmodified subroutine.

** E.g., diagonal entries are a comparison of unmodified and modified subroutine generated values for covariance.

*** E.g., off-diagonal entries are a comparison of generated values for covariance between parameters in intersecting rows and columns.

APPENDIX J

MODIFICATION OF STHESIS

APPENDIX J

MODIFICATION OF STHESIS

The Downs and Forseth program STHESIS was modified so that correlated release parameters could be used to generate simulated bomb impacts. An explanation of the modifications, by line number, are provided as follows:

HOUSEKEEPING SECTION

Line 120	Dimension statement for the variance-covariance matrix (VPAR), vector of means (EXPAR), and the resulting release parameters vector (XPAR) provided by the MVN subroutine.

MEAN/VARIANCE-COVARIANCE MATRIX

Lines 1550-1620	Program reads the vector of means and the variance-covariance matrix.

PERFECT BOMB

Line 1660	Logic statement which provides for assigning the initial values of the six release parameters dependent upon the input file format.

DISTRIBUTION SELECTION LOGIC

Line 2070 — Logic statement which circumvents generation of independent release parameters if MVN input file used.

GENERATE CORRELATED RELEASE PARAMETERS

Lines 2470-2550 — Call statement for the multivariate normal subroutine (MVN) to provide correlated release parameters. The correlated release parameter values are then assigned to the appropriate variable names for computation of the resulting bomb impact.

CORRELATED RELEASE PARAMETERS SUBROUTINE

Lines 6550-7080 — This subroutine calculates the correlated release parameters which are passed back to the main program. Definitions of the subroutine arguments are also provided.

NORMAL DISTRIBUTION SUBROUTINE

Lines 7100-7270 — This normal distribution subroutine is used with the MVN subroutine to generate the normally distributed, correlated release parameters. Definitions of the subroutine arguments are also provided.

```
10C       ****DOWNS/FORSETH BOMBING MODEL****
20C       ****BROWN/CALLEN MODIFICATION****
30C
40C       ***HOUSEKEEPING SECTION***
50C
60C        *PROGRAM REQUIRES FILES 13 AND 14 *
70C        *IN THIS PROGRAM THE FILES ARE "CALL ATTACHED" *
80         IMPLICIT REAL(K-Z)
90         CHARACTER COMMENT1*20,COMMENT2*20,COMMENT3*20
100        CHARACTER BLNAME*10
110        COMMON SEED,PARAM(20,7)
120        DIMENSION VPAR(6,6), EXPAR(6), XPAR(6)
130        DIMENSION PRODATA(10)
140        CALL ATTACH(13,"77A80/DATANB;",3,0,,)
150        CALL ATTACH(14,"77A80/RAWDATA;",3,0,,)
160C
170C       **HEADING**
180        PRINT,""
190        PRINT,""
200        PRINT,"DOWNS/FORSETH BOMB DISTRIBUTION MODEL"
210        PRINT,"      BROWN/CALLEN MODIFICATION"
220        PRINT,""
230        PRINT,""
240C
250C       ***CONTROL PARAMETERS***
260C
270C       **CONTROL PARAMETERS READ**
```

Figure J.1 STHENEW Program Listing

```
      READ(13,0005)ILN,BLNAME
 0005 FORMAT(I3,R10)
      DO 0015 I=1,9
      READ(13,0010)ILN,PRODATA(I)
 0010 FORMAT(V)
 0015 CONTINUE
      TYPE=PRODATA(1)
      LAT=PRODATA(2)
      TARALT=PRODATA(3)
      VELOCITY=PRODATA(4)
      TDELAY=PRODATA(5)
      NI=PRODATA(6)
      SEED=PRODATA(7)
      TIME=PRODATA(8)
      CELLSIZE=PRODATA(9)
C
C     **CONTROL PARAMETERS PRINTOUT**
      WRITE(14)NI,CELLSIZE
      PRINT 0018,BLNAME
 0018 FORMAT(1X,"INPUT FILE IS - ",25X,R10,//)
      PRINT,"** CONTROL PARAMETERS **"
      IF(TYPE-2.0) ,0022,0025
      PRINT,"TYPE OF BOMB DROPPED -                      MK-106"
      GO TO 0030
 0022 PRINT,"TYPE OF BOMB DROPPED -                    BDU-33 A/B"
      GO TO 0030
 0025 PRINT,"TYPE OF BOMB DROPPED -                      MK-82"
 0030 PRINT 0035,LAT
 0035 FORMAT(1X,"LATITUDE OF TARGET IS -DEGREES- ",12X,F9.5)
      PRINT 0037,TARALT
 0037 FORMAT(1X,"ALTITUDE OF TARGET -FEET- ",16X,F11.5)
      PRINT 0038,TDELAY
```

Figure J.1 (continued)

```
0038 FORMAT(1X,"AIRCRAFT SYSTEM TIME DELAY -SECONDS- ",7X,F9.5)
     PRINT 0039,VELOCITY
0039 FORMAT(1X,"BOMB EJECTION VELOCITY -FEET PER SECOND- ",3X,F9.5)
     PRINT 0040,NI
0040 FORMAT(1X,"NUMBER OF BOMBS DROPPED - ",14X,F8.0,//)
     PRINT 0050,SEED
0050 FORMAT(1X,"SEED VALUE - ",29X,F6.0)
     PRINT 0060,TIME/10.0
0060 FORMAT(1X,"ITERATION TIME APPROX -SECONDS- ",12X,F9.5)
     PRINT 0065,CELLSIZE
0065 FORMAT(1X,"INITIAL CELLSIZE -FEET- ",19X,F5.0,///)
C
C    ***BOMB SELECTION***
C
0070 IF(TYPE-2.0)0080,0090,0100
C
C    **MK-106 BOMB DRAG EQUATION VALUES**
0080 N0=.1110206423
     N1=-.2367024387
     N2=.1404399922
     N3=0.0
     N4=0.0
     N5=0.0
     N6=0.0
     D0=.4444444444
     D1=-.9501149716
     D2=.5337518871
     D3=.1912320558
     D4=-.3010985452
     D5=.1237935001
     D6=-.0168992064
     C1C=5.866666663
```

Figure J.1 (continued)

```
920       A=3.87
930       W=4.63
940       GO TO 0110
950C
960C      **BDU-33 A/B OR B/B BOMB DRAG EQUATION VALUES**
970 0090 N0=139
980       N1=-486.2
990       N2=594
1000       N3=-121
1010       N4=-328
1020       N5=231
1030       N6=-27
1040       D0=1000
1050       D1=-3581
1060       D2=5083
1070       D3=-3416
1080       D4=918
1090       D5=0.0
1100       D6=0.0
1110       C1C=1.0
1120       A=4.0
1130       W=23.6
1140       GO TO 0110
1150C
1160C      **MK-82 BOMB DRAG EQUATION VALUES**
1170 0100 N0=.111437408
1180       N1=-.237786173
1190       N2=.129739387
1200       N3=0.0
1210       N4=0.0
1220       N5=0.0
1230       N6=0.0
```

Figure J.1 (continued)

```
      D0=.386536759
      D1=-.952380955
      D2=.788346071
      D3=-.245982881
      D4=.025483012
      D5=.0031222
      D6=0.0
      C1C=.607364168
      A=10.75
      W=500
C
C     ***RELEASE PARAMETERS READ/PRINTOUT***
C
0110  DO 0120 I=1,6
      READ(13,0010)ILN,(PARAM(I,J),J=1,7)
0120  CONTINUE
      PRINT,"** RELEASE PARAMETERS **"
      PRINT,"RELEASE ALTITUDE STRING OF--"
      PRINT 0125,(PARAM(1,J),J=1,7)
0125  FORMAT(1X,7F10.2)
      PRINT,"RELEASE TRUE AIR SPEED STRING OF--"
      PRINT 0125,(PARAM(2,J),J=1,7)
      PRINT,"RELEASE PITCH (+ IS UP) STRING OF--"
      PRINT 0125,(PARAM(3,J),J=1,7)
      PRINT,"RELEASE LEAD OR TRAIL STRING OF--"
      PRINT 0125,(PARAM(4,J),J=1,7)
      PRINT,"RELEASE HEADING STRING OF--"
      PRINT 0125,(PARAM(5,J),J=1,7)
      PRINT,"RELEASE OFFSET (+ IS RIGHT) STRING OF--"
      PRINT 0125,(PARAM(6,J),J=1,7)
C
C     **MEAN/VARIANCE-COVARIANCE MATRIX**
```

Figure J.1 (continued)

```
C
      DO 126 I=1,6
      EXPAR(I)=PARAM(I,1)
      DO 126 J=2,7
126   VPAR(I,J-1)=PARAM(I,J)
      XK5=1.0
      J=6
C
C     ***PERFECT BOMB***
C
      IF(BLNAME.EQ." MVN        ")  J=1
      Z=PARAM(1,J)
      C=PARAM(2,J)
      ZANGLE=PARAM(3,J)
      RAOFFSET=PARAM(4,J)
      XYANGLE=PARAM(5,J)
      DEOFFSET=PARAM(6,J)
      PRINT,""
      PRINT,""
      PRINT,"** PERFECT BOMB **"
      PRINT," * INPUT VALUES *"
      PRINT,"RELEASE PARAMETERS(AFFECTING RANGE ERROR)ARE-"
      PRINT,"ALTITUDE   TRUE AIR SPEED   PITCH   LEAD/TRAIL   HEADING"
      PRINT,"  FEET          KNOTS        DEGREES      FEET       DEGREES"
      PRINT 0130,Z,C,F,RAOFFSET,XYANGLE
0130  FORMAT(1X,F7.0,8X,F5.0,7X,F6.2,F12.2,F12.2,//)
      PRINT,"RELEASE PARAMETERS(AFFECTING DEFLECTION ERROR)ARE-"
      PRINT,"HEADING   OFFSET"
      PRINT,"DEGREES    FEET"
      PRINT 0140,XYANGLE,DEOFFSET
0140  FORMAT(1X,F7.2,F9.2)
      PRINT," "
```

Figure J.1 (continued)

```
C     PRINT," * OUTPUT VALUES *"
      PRINT,"ACTUAL TIME OF FALL       TRAIL       RANGE"
      PRINT,"      SECONDS              FEET        FEET"
C
C     ***SET SEA LEVEL GRAVITY***
C
      LAT=LAT*.0174532925199
      SEAGRAV=1+.0052885*SIN(LAT)**2-.0000059*SIN(2.*LAT)**2
      SEAGRAV=SEAGRAV*9.780356*3.280839895013
C
C     ***DROP BOMBS***
C
      INI=NI+1.0
      DO 450 IIII=1,INI
      IF(IIII-1)330,330,
C
C     ***DISTRIBUTION SELECTION LOGIC***
C
C     **SELECT DISTRIBUTION AND GENERATE RANDOM NUMBER**
      IF(BLNAME.EQ." MVN       ") GO TO 325
      DO 320 J=1,6
      I=J
      GO TO(150,160,170,180,190,200,210,220,230,240),PARAM(I,7)
  150 XNUM=RNORM(I)
      GO TO 250
  160 XNUM=UNFRM(PARAM(I,2),PARAM(I,3))
      GO TO 250
  170 XNUM=RLOGN(I)
      GO TO 250
  180 XNUM=ERLNG(I)
      GO TO 250
  190 XNUM=NPSSN(I)
```

Figure J.1 (continued)

```
      GO TO 250
  200 XNUM=BETA(I)
      GO TO 250
  210 XNUM=GAMA(I)
      GO TO 250
  220 XNUM=WEIB(I)
      GO TO 250
  230 XNUM=CAUCHY(I)
      GO TO 250
  240 XNUM=PARAM(I,6)
C
C     **INPUT RANDOM NUMBER INTO RELEASE PARAMETER VALUE**
  250 GO TO(260,270,280,290,300,310),I
  260 Z=XNUM
      GO TO 320
  270 C=XNUM
      GO TO 320
  280 ZANGLE=XNUM
      GO TO 320
  290 RANGEOFF=XNUM
      GO TO 320
  300 XYANGLE=XNUM
      GO TO 320
  310 DEFLEOFF=XNUM
  320 CONTINUE
      GO TO 330
C
      **GENERATE CORRELATED RELEASE PARAMETERS IF MVN INPUT**
 325  CALL MVN(6,XK5,VPAR,EXPAR,XPAR)
      XK5=XK5+1.0
      Z=XPAR(1)
      C=XPAR(2)
```

Figure J.1 (continued)

```
      ZANGLE=XPAR(3)
      RANGEOFF=XPAR(4)
      XYANGLE=XPAR(5)
      DEFLEOFF=XPAR(6)
C
C     ***BALLISTIC PATH COMPUTATION***
C
C     **SET AND CONVERT VALUES**
  330 E=VELOCITY
      ZANGLE=ZANGLE*.017453292519943
      XYANGLE=XYANGLE*.017453292519943
      C=C*1.6878
      ACGNDVEC=C*COS(ZANGLE)
      X0=ACGNDVEC*COS(XYANGLE)
      Y0=ACGNDVEC*SIN(XYANGLE)
      Z0=C*SIN(ZANGLE)
C
C     **SET COORDINATES UPON BOMB RELEASE SIGNAL GENERATION**
      X=X0*TDELAY
      Y=Y0*TDELAY
      Z=Z+Z0*TDELAY
      F=F+TDELAY
      BMALTREM=Z-TARALT
      BMGNDVEC=ACGNDVEC+E*COS(ZANGLE-1.570796327)
      X1=BMGNDVEC*COS(XYANGLE)
      Y1=BMGNDVEC*SIN(XYANGLE)
      Z1=C*SIN(ZANGLE)+E*SIN(ZANGLE-1.570796327)
C
C     **ITERATIVE PROCESS (FUNCTION OF TIME)**
  340 C=SQRT(X1**2+Z1**2+Y1**2)
      CALL ATMOS(Z,DUM1,DUM2,DUM3,DUM4,DUM5,SOUND,DUM7,DUM8)
      M=C/SOUND
```

Figure J.1 (continued)

```
      C1=N0+N1*M+N2*M**2+N3*M**3+N4*M**4+N5*M**5+N6*M**6
      C1=C1C*C1/(D0+D1*M+D2*M**2+D3*M**3+D4*M**4+D5*M**5+D6*M**6)
      K=C1*C**2*(-2.085536E-04)*A**2/W
      K=K*(1-(6.87535E-06)*Z)**4.2561
      IF(TYPE-2.0) ,360,360
      IF(F-1.0) , ,360
  350 K=K*1.2
  360 ZANGLE=ATAN(Z1/(SQRT(X1**2+Y1**2)))
      XYANGLE=ATAN(Y1/X1)
      X2=K*COS(ZANGLE)*COS(XYANGLE)
      Y2=K*COS(ZANGLE)*SIN(XYANGLE)
      Z2=K*SIN(ZANGLE)
      T=TIME/(-K)
      IF(Z1) ,400,
      T1=BMALTREM/(-Z1)
      IF(T1)400, ,
      IF(T1.GT.T)GO TO 400
      T=T1
  400 DELTAX=0.5*X2*T**2+X1*T
      X=X+DELTAX
      X1=X1+X2*T
      DELTAY=0.5*Y2*T**2+Y1*T
      Y=Y+DELTAY
      Y1=Y1+Y2*T
      GRAVITY=SEAGRAV-(.30783368E-05)*Z
      DELTAZ=0.5*(Z2-GRAVITY)*T**2+Z1*T
      BMALTREM=BMALTREM+DELTAZ
      Z1=Z1+(Z2-GRAVITY)*T
      F=F+T
      IF(BMALTREM) ,410,340
C
C     **LAST ITERATION CORRECTION**
```

Figure J.1 (continued)

```
C
      TCORR=BMALTREM/Z1
      F=F-TCORR
      X=X-X1*TCORR
      Y=Y-Y1*TCORR
C
C     **BOMB DROP SUMMARY DATA**
  410 F=F+TDELAY
      X=X+RANGEOFF-XRANGE
      Y=Y+DEFLEOFF-YOFFSET
C
C     **END ACTUAL BALLISTIC PATH COMPUTATION**
C
C     **PERFECT BOMB IMPACT DATA AND PRINTOUT**
      IF(BESTBOMB-1.0) ,430,430
      ATF=F
      XRANGE=X+RAOFFSET
      YOFFSET=Y+DEOFFSET
      TRAIL=X0*F-XRANGE
      PRINT 420,ATF,TRAIL,XRANGE
  420 FORMAT(1X,F12.2,9X,F9.0,F11.0,//)
      BESTBOMB=1.0
      GO TO 450
C
C      **SAMPLE BOMB CUMMULATIVE DATA**
  430 CE=SQRT(X**2+Y**2)
      SUMCE=SUMCE+CE
      SUMCECE=SUMCECE+CE**2
      SUMX=SUMX+X
      SUMXX=SUMXX+X**2
      SUMY=SUMY+Y
      SUMYY=SUMYY+Y**2
C
```

Figure J.1 (continued)

```
C     ***MAXIMUM/MINIMUM IMPACT POINTS ***
C
      XM=MIN(X,XM)
      YM=MIN(Y,YM)
      CEM=MIN(CE,CEM)
      XGT=MAX(X,XGT)
      YGT=MAX(Y,YGT)
      CEGT=MAX(CE,CEGT)
C
C     **SAMPLE BOMB IMPACT DATA WRITE TO RAWDATA FILE**
      WRITE(14)X,Y
      DROPS=DROPS+1.0
  450 CONTINUE
C
C     ***PRINT IMPACT SUMMARY DATA***
C
      CEMEAN=SUMCE/NI
      CESD=SQRT((NI*SUMCECE-SUMCE**2)/(NI*(NI-1.)))
      DEMEAN=SUMY/NI
      DEFLECSD=SQRT((NI*SUMYY-SUMY**2)/(NI*(NI-1.)))
      RAMEAN=SUMX/NI
      RANGESD=SQRT((NI*SUMXX-SUMX**2)/(NI*(NI-1.)))
      PRINT,"** IMPACT SUMMARY DATA **"
      PRINT,"                    MEAN  STD DEV  LARGEST  SMALLEST"
      PRINT 460,RAMEAN,RANGESD,XGT,XM
  460 FORMAT(1X,"RANGE",10X,F6.1,F8.1,F9.1,F10.1)
      PRINT 470,DEMEAN,DEFLECSD,YGT,YM
  470 FORMAT(1X,"DEFLECTION",5X,F6.1,F8.1,F9.1,F10.1)
      PRINT 480,CEMEAN,CESD,CEGT,CEM
  480 FORMAT(1X,"CIRCULAR ERROR",1X,F6.1,F8.1,F9.1,F10.1)
C
C     **WRITE DATA TO RAWDATA FILE FOR PLOT OUTPUT**
```

Figure J.1 (continued)

```
      WRITE(14)TYPE
      WRITE(14)PARAM(1,6)
      WRITE(14)PARAM(2,6)
      WRITE(14)PARAM(3,6)
      READ(13,495,END=497)LN,COMMENT1
      READ(13,495,END=497)LN,COMMENT2
      READ(13,495,END=497)LN,COMMENT3
  495 FORMAT(I4,R20)
  497 WRITE(14)COMMENT1
      WRITE(14)COMMENT2
      WRITE(14)COMMENT3
      WRITE(14)DEMEAN,RAMEAN
      WRITE(14)DEFLECSD,RANGESD
C
C     **NUMBER OF BOMB DROPS DIAGNOSTIC**
      IF(DROPS-NI) ,500,
      PRINT,"PROGRAM ERROR **SOME DROPS NOT MADE**"
      PRINT,"***ALL OUTPUT BAD - RERUN***"
C
C     ***ENDS PRINTOUTS AND FILES***
C
  500 REWIND 13
      REWIND 14
      STOP
      END
C
C
C     ** DISTRIBUTION FUNCTIONS **
C
      FUNCTION RNORM (J)
      COMMON SEED,PARAM(20,7)
    5 RNORM=PARAM(J,4)*((-2*ALOG(RND(SEED)))**.5*COS(6.283*
```

Figure J.1 (continued)

```
&      RND(SEED)))+PARAM(J,1)
       IF (RNORM -PARAM (J,2)) 5,7,8
     7 RETURN
     8 IF (RNORM -PARAM (J,3)) 7,7,5
       END
C
       FUNCTION UNFRM (A,B)
       COMMON SEED,PARAM(20,7)
       UNFRM=A+(B-A)*RND(SEED)
       RETURN
       END
C
       FUNCTION RLOGN(J)
       COMMON SEED,PARAM(20,7)
       PARAM(20,1)=PARAM(J,1)
       PARAM(20,2)=PARAM(J,1)-4.*PARAM(J,4)
       PARAM(20,3)=PARAM(J,1)+4.*PARAM(J,4)
       PARAM(20,4)=PARAM(J,4)
       VA=RNORM(20)
       RLOGN=EXP(VA)
       SF=EXP(PARAM(J,1)+4*PARAM(J,4))
       RLOGN=(RLOGN/SF)*(PARAM(J,3)-PARAM(J,2))+PARAM(J,2)
       RETURN
       END
C
       FUNCTION ERLNG (J)
       COMMON SEED,PARAM(20,7)
       JJ=PARAM(J,4)
       IF(JJ-1.)8,9,10
     8 PRINT,"THAT IS AN ERLNG ERROR"
       STOP
     9 ERLNG=-PARAM(J,1)*ALOG(RND(SEED))
```

Figure J.1 (continued)

```
      GO TO 11
   10 ERLNG=0.
      DO 2 I=1,JJ
    2 ERLNG=ERLNG-PARAM(J,1)*ALOG(RND(SEED))
   11 IF(ERLNG-PARAM(J,2))7,6,6
    7 ERLNG=PARAM(J,2)
    5 RETURN
    6 IF(ERLNG - PARAM(J,3))5,5,4
    4 ERLNG=PARAM(J,3)
      RETURN
      END
C
      FUNCTION NPSSN(J)
      COMMON SEED,PARAM(20,7)
      NPSSN=0
      P=PARAM(J,1)
    1 IF(P-6.0)2,2,4
    2 Y=EXP(-P)
      X=1.0
    3 Z=RND(SEED)
      X=X*Z
      IF(X-Y)6,8,8
    8 NPSSN=NPSSN+1
      GO TO 3
    4 TEMP=PARAM(J,4)
      PARAM(J,4) = (PARAM(J,1))**.5
      NPSSN=RNORM(J)
      PARAM (J,4)=TEMP
      IF(NPSSN)4,6,6
    6 KK=PARAM(J,2)
      KKK=PARAM (J,3)
      NPSSN=KK+NPSSN
```

Figure J.1 (continued)

```
      IF(NPSSN-KKK)7,7,9
    9 NPSSN=PARAM(J,3)
    7 RETURN
      END
C
      FUNCTION WEIB(J)
      COMMON SEED,PARAM(20,7)
    1 WK=PARAM(J,1)
      WM=PARAM(J,2)
      WEIB=((-(WM+1.)/WK)*ALOG(RND(SEED)))**(1./(WM+1.))
      IF(WEIB-PARAM(J,3))1,2,2
    2 IF(WEIB-PARAM(J,4))3,3,1
    3 RETURN
      END
C
      FUNCTION GAMA(J)
      COMMON SEED,PARAM(20,7)
      A=PARAM(J,4)+1.0
      B=PARAM(J,1)
      GAMA=GAM(A)*B
      SF=B*A+4.*(((B**2.)*A)**.5)
      GAMA=(GAMA/SF)*(PARAM(J,3)-PARAM(J,2))+PARAM(J,2)
      RETURN
      END
      FUNCTION GAM(AK)
      COMMON SEED,PARAM(20,7)
      K=AK
      FK=K
      GAM=0.
      IF (K)103,103,101
  101 PROD=1.0
      DO 102 I=1,K
```

Figure J.1 (continued)

```
  102 PROD=PROD*RND(SEED)
      GAM=-ALOG(PROD)
  103 DG=AK-FK
      IF (DG-.015)110,110,104
  104 IF (DG-.985)106,105,105
  105 W=1.
      GO TO 109
  106 A=1./DG
      B=1./(1.-DG)
  107 X=RND(SEED)**A
      Y=RND(SEED)**B+X
      IF (Y-1.)108,108,107
  108 W=X/Y
  109 Y=-ALOG(RND(SEED))
      GAM=GAM+W*Y
  110 RETURN
      END
C
      FUNCTION BETA(J)
      COMMON SEED,PARAM(20,7)
      A=PARAM(J,4)+1.0
      B=PARAM(J,1)
      X=GAM(A)
      BETA=X/(X+GAM(B))
      BETA=BETA*(PARAM(J,3)-PARAM(J,2))+PARAM(J,2)
      RETURN
      END
C
      FUNCTION CAUCHY(J)
      COMMON SEED,PARAM(20,7)
      THETA=PARAM(J,1)
      LAMBDA=PARAM(J,4)
```

Figure J.1 (continued)

```
    1 RAD=3.1415927*(RND(SEED)-.5)
      CAUCHY=LAMBDA*(SIN(RAD)/COS(RAD))+THETA
      IF(CAUCHY-PARAM(J,2))1,2,2
    2 IF(CAUCHY-PARAM(J,3))3,3,1
    3 RETURN
      END
C
C
C     ***RANDOM NUMBER GENERATOR***
      FUNCTION RND(X)
      DATA IA,SMALL,IBIG/190979,.291038305E-10,34359738367/
      IF(J.NE.0)GO TO 25
      KX=X
      IF(KX),15,5
      CALL YTIME(KX)
      CALL YDATE(KKX)
      KX=KX+KKX
    5 IF(KX.GE.185389)GO TO 12
      X=KX
      FACTOR=185389/X+1.0001
      KX=X*FACTOR
   12 IF(MOD(KX,5).EQ.0)KX=KX-2
      IF(MOD(KX,2).EQ.0)KX=KX+1
      GO TO 20
   15 KX=357495
   20 J=1
   25 KX=KX*IA
      IF(KX),30,30
      KX=KX+IBIG+1
   30 Y=KX
   35 RND=Y*SMALL
      RETURN
```

Figure J.1 (continued)

```
5720        END
5730C
5740C       ***SPEED OF SOUND COMPUTATION SUBROUTINE***
5750        SUBROUTINE ATMOS (ZFT,TM,SIGMA,RHO,THETA,DELTA,CA,AMU,K)
5760C       THIS IS A SUBROUTINE TO COMPUTE CERTAIN ELEMENTS OF THE 1962
5770C       U.S. STANDARD ATMOSPHERE UP TO 90 KILOMETERS.
5780C       CALLING SEQUENCE...
5790C
5800C       CALL ATMOS (ZFT, TM, SIGMA, RHO, THETA, DELTA, CA, AMU, K)
5810C            ZFT    =   GEOMETRIC ALTITUDE (FEET)
5820C            TM     =   MOLECULAR SCALE TEMPERATURE (DEGREES RANKINE)
5830C            SIGMA  =   RATIO OF DENSITY TO THAT AT SEA LEVEL
5840C            RHO    =   DENSITY (LB-SEC**2-FT**(-4)   OR   SLUGS-FT**3)
5850C            THETA  =   RATIO OF TEMPERATURE TO THAT AT SEA LEVEL
5860C            DELTA  =   RATIO OF PRESSURE TO THAT AT SEA LEVEL
5870C            CA     =   SPEED OF SOUND (FT/SEC)
5880C            AMU    =   VISCOSITY COEFFICIENT (LB-SEC/FT**2)
5890C
5900C            K = 1 NORMAL
5910C              = 2 ALTITUDE LESS THAN -5000 METERS OR GREATER THAN 90 KM
5920C              = 3 FLOATING POINT OVERFLOW
5930C
5940C       ALL DATA AND FUNDAMENTAL CONSTANTS ARE IN THE METRIC SYSTEM AS
5950C       THESE QUANTITIES ARE DEFINED AS EXACT IN THIS SYSTEM.
5960C
5970C       THE RADIUS OF THE EARTH (REFT59) IS THE VALUE ASSOCIATED WITH THE
5980C       1959 ARDC ATMOSPHERE SO THAT PROGRAMS CURRENTLY USING THE LIBRARY
5990C       ROUTINE WILL NOT REQUIRE ALTERATION TO USE THIS ROUTINE.
6000        DIMENSION HB(10),TMB(10),DELTAB(10),ALM(10)
6010        DATA(HB(I),I=1,10)/-5.,0.,11.,20.,32.,47.,52.,61.,79.,88.743/
6020        DATA(TMB(I),I=1,10)/320.65,288.15,216.65,216.65,228.65,270.65,
6030&       270.65,252.65,180.65,180.65/
```

Figure J.1 (continued)

```
&     DATA(DELTAB(I),I=1,10)/1.75363,1.,2.23361E-01,5.40328E-02,
&     8.56663E-03,1.09455E-03,5.82289E-04,1.79718E-04,1.0241E-05,
&     1.6223E-06/
      DATA(ALM(I),I=1,10)/-6.5,-6.5,0.,1.,2.8,0.,-2.,-4.,0.,0./
      DATA  REFT59/2.0855531E 07/, GZ     /9.80665/,
&           AMZ    /28.9644       /, RSTAR /8.31432/,
&           FTTOKM/3.048E-04     /,S      /110.4  /,
&           AMUZ   /1.2024E-05    /, CAZ   /1116.45/,
&           RHOZ   /0.076474      /, GZENG /32.1741/
C     CONVERT GEOMETRIC ALTITUDE TO GEOPOTENTIAL ALTITUDE
      HFT = (REFT59/(REFT59+ZFT))*ZFT
C     CONVERT HFT AND ZFT TO KILOMETERS
      Z = FTTOKM*ZFT
      H = FTTOKM*HFT
      K = 1
      TMZ = TMB(2)
      IF(H+5.0)16, ,
      IF(H-90.0) , ,16
      DO 10 M=1,10
      IF (H-HB(M)) 11,12,10
   10 CONTINUE
      GO TO 16
   11 M = M-1
   12 DELH = H-HB(M)
      IF(ALM(M)-0.0) ,13,
      TMK = TMB(M)+ALM(M)*DELH
C     GRADIENT IS NON ZERO, PAGE 10, EQUATION I.2.10-(3)
      DELTA = DELTAB(M)*((TMB(M)/TMK)**(GZ*AMZ/(RSTAR*ALM(M))))
      GO TO 14
   13 TMK = TMB(M)
C     GRADIENT IS ZERO, PAGE 10, EQUATION I.2.10-(4)
      DELTA = DELTAB(M)*EXP(-GZ*AMZ*DELH/(RSTAR*TMB(M)))
```

Figure J.1 (continued)

```
 14 THETA = TMK/TMZ
    SIGMA = DELTA/THETA
    ALPHA = SQRT(THETA**3)*((TMZ+S)/(TMK+S))
    CONVERSION TO ENGLISH UNITS
    TM = 1.8*TMK
    RHO = RHOZ*SIGMA/GZENG
    CA = CAZ*SQRT(THETA)
    AMU = AMUZ*ALPHA/GZENG
    CALL OVERFL(J)
    J=2
    GO TO (15,17), J
 15 K = K+2
    GO TO 17
 16 K = 2
    PRINT,"ALTITUDE NOT WITHIN LIMITS (-5 TO 90 KM)"
    PRINT,"*****************************************"
 17 RETURN
    END

    ***CORRELATED RELEASE PARAMETERS SUBROUTINE***

    SUBROUTINE MVN(N,XK5,V,EX,X)
    THIS SUBROUTINE GENERATES "N" RANDOM VARIABLES FROM A
    MULTIVARIATE NORMAL DISTRIBUTION AS DEFINED BY A VECTOR
    OF MEANS AND A VARIANCE-COVARIANCE MATRIX.  THEORETICAL
    ASPECTS OF THE SUBROUTINE MAY BE FOUND IN THE
    REFERENCE--COMPUTER SIMULATION EXPERIMENTS WITH MODELS OF
    ECONOMIC SYSTEMS BY THOMAS H. NAYLOR, PG.396FF.
    DEFINITION OF SUBROUTINE ARGUMENTS:
         N = NUMBER OF RELEASE PARAMETERS
         XK5 = PROGRAM CONTROL PARAMETER SET EQUAL TO ONE FIRST
         TIME MVN CALLED, NOT EQUAL TO ONE THEREAFTER.
```

Figure J.1 (continued)

```
C          V = VARIANCE-COVARIANCE MATRIX
C          EX = MEANS' VECTOR
C          X = GENERATED RELEASE PARAMETERS PASSED BACK TO MAIN PROGRAM
      DIMENSION V(6,6), EX(6), X(6), D(6,6), Z(6), SUM2(6)
      K5=XK5
      IF (K5-1) 4,4,29
    4 DO 7 J1=1,N
      X(J1)=0.0
      DO 7 J2=1,N
    7 D(J1,J2)=0.0
      DO 9 I=1,N
    9 D(I,1)=V(I,1)/V(1,1)**.5
      DO 28 I=2,N
      SUM=0.0
      K1=I-1
      DO 14 K=1,K1
   14 SUM=SUM+D(I,K)*D(I,K)
      CK=V(I,I)-SUM
      IF (CK) 17,17,18
   17 STOP
   18 D(I,I)=SQRT(CK)
      IF(I-N) 20,28,28
   20 K1=I
      DO 27 J=2,K1
      SUM1=0.0
      K2=J-1
      DO 25 K=1,K2
   25 SUM1=SUM1+D(I+1,K)*D(J,K)
      D(I+1,J)=(V(I+1,J)-SUM1)/D(J,J)
   27 CONTINUE
   28 CONTINUE
   29 DO 31 I=1,N
```

Figure J.1 (continued)

```
      SUM2(I)=0.0
   31 CALL NORMAL (0.0, 1.0,Z(I))
      DO 34 I=1,N
      DO 34 J=1,N
   34 SUM2(I)=SUM2(I)+D(I,J)*Z(J)
      DO 36 I=1,N
   36 X(I)=SUM2(I)+EX(I)
      RETURN
      END
C
C     **NORMAL DISTRIBUTION SUBROUTINE**
C
      SUBROUTINE NORMAL (EX,STDX,X)
C     THIS SUBROUTINE IS USED BY SUBROUTINE MVN TO GENERATE
C     SINGLE RANDOM VARIABLES FROM A NORMAL DISTRIBUTION.
C     DEFINITION OF SUBROUTINE ARGUMENTS:
C          EX = MEAN
C          STDX = STANDARD DEVIATION
C          X = GENERATED RANDOM VARIABLE PASSED BACK TO
C          SUBROUTINE MVN
      COMMON SEED,PARAM(20,7)
      SUM=0.0
      DO 4 I=1,12
      R=RND(SEED)
    4 SUM=SUM+R
      X=STDX*(SUM-6.0)+EX
      RETURN
      END
```

Figure J.1 (continued)

APPENDIX K

STATISTICAL AND GRAPHICAL RESULTS

TABLE K.1

PROFILE B

INDEPENDENT RELEASE PARAMETERS

Input Release Parameters

	μ	Min	Max	σ	Desired
Altitude:	3745.39	3584.00	3917.00	82.20	3745.39
Airspeed:	282.38	272.80	299.20	6.68	282.38
Pitch:	-30.99	-34.80	-28.60	1.41	-30.99
Lead:	8.83	-299.00	310.00	146.75	8.83
Heading:	6.20	2.59	10.88	1.82	6.20
Offset:	-212.78	-353.00	-34.00	70.09	-212.78

Independence Check

Number of Cells: 8 χ^2 Critical Value: 7.815

Degrees of Freedom: 3 χ^2 Sample Value: 19.903

Statistical Analysis (10,000 Bombs)

	Min	Max	$\bar{x}$	s	χ^2 Crit	χ^2 Sample
X	-560.4	632.5	-2.9	168.7	19.6750	28.4559
Y	-266.3	339.0	5.0	89.9	(Failed K-S Test)	
CE	0	650.5	167.4	92.4		

Conclusion: X is not normally distributed at the 95% level.

Y did not fit any of the distributions tested by SSIMFIT.

Therefore, the distribution is not bivariate normal.

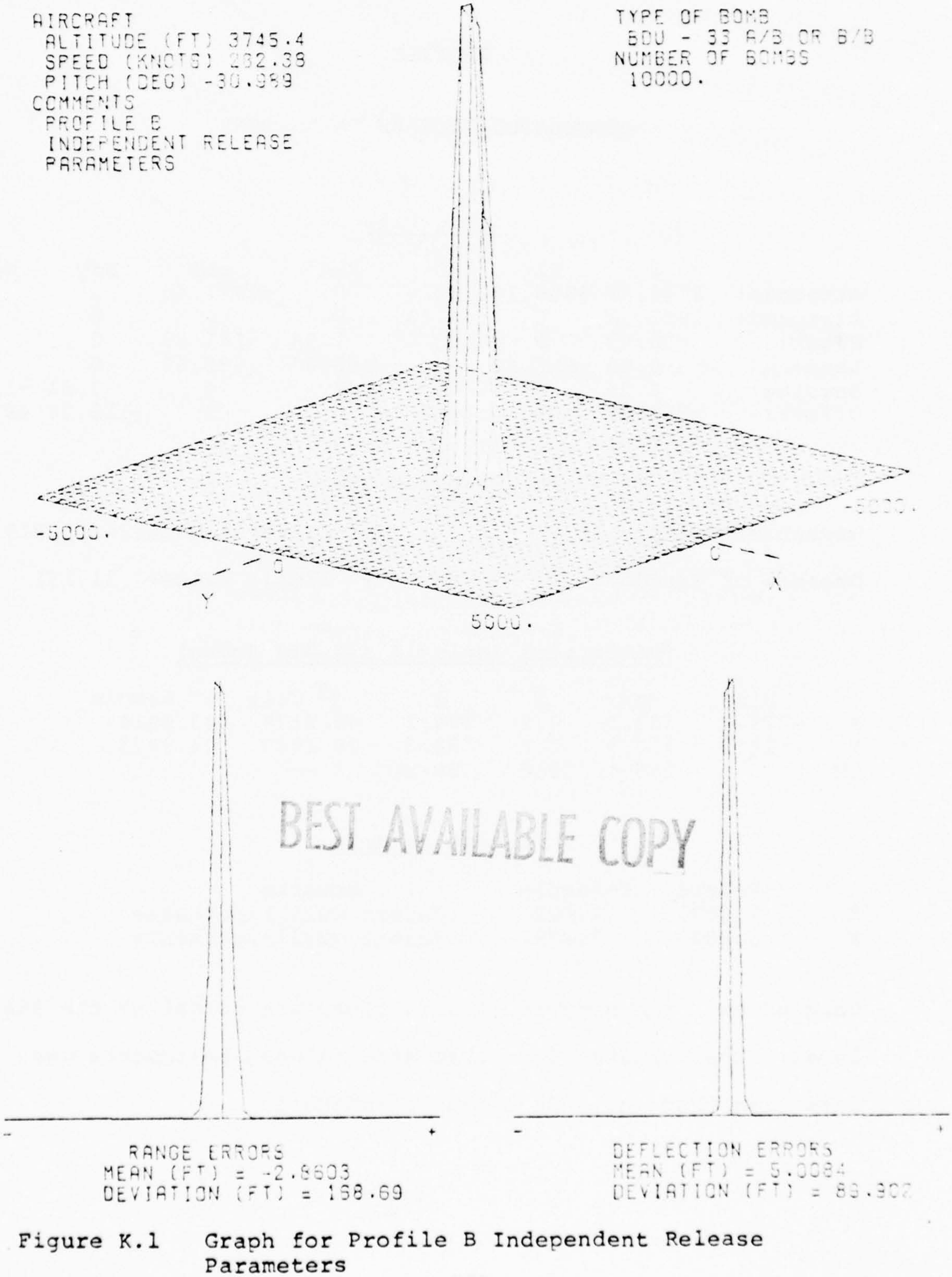


Figure K.1 Graph for Profile B Independent Release Parameters

TABLE K.2

PROFILE B

CORRELATED RELEASE PARAMETERS

Input File

	μ	Alt	CAS	FPA	Lead	Hdg	Offset
Altitude:	3745.39	6756.13	0	0	-8017.40	0	0
Airspeed:	282.38	0	44.64	0	0	0	0
Pitch:	-30.99	0	0	1.98	-163.83	0	0
Lead:	8.83	-8017.40	0	-163.83	21535.68	0	0
Heading:	6.20	0	0	0	0	3.31	-115.28
Offset:	-212.78	0	0	0	0	-115.28	4913.00

Independence Check

Number of Cells: 16 χ^2 Critical Value: 16.919

Degrees of Freedom: 9 χ^2 Sample Value: 11.152

Statistical Analysis (10,000 Bombs)

	Min	Max	$\bar{x}$	s	χ^2 Crit	χ^2 Sample
X	-375.9	363.0	0.2	104.2	48.3179	43.3914
Y	-118.9	116.8	0.6	33.3	30.1440	24.9873
CE	0	376.1	92.0	59.1	--	--

F-test Results

	F-Crit	F-Sample	Remarks
X	1.000	2.622	Reject null hypothesis
Y	1.000	7.279	Reject null hypothesis

Conclusion: The distribution is bivariate normal at the 95% level. The variance for correlated release parameters was less than that for independent parameters.

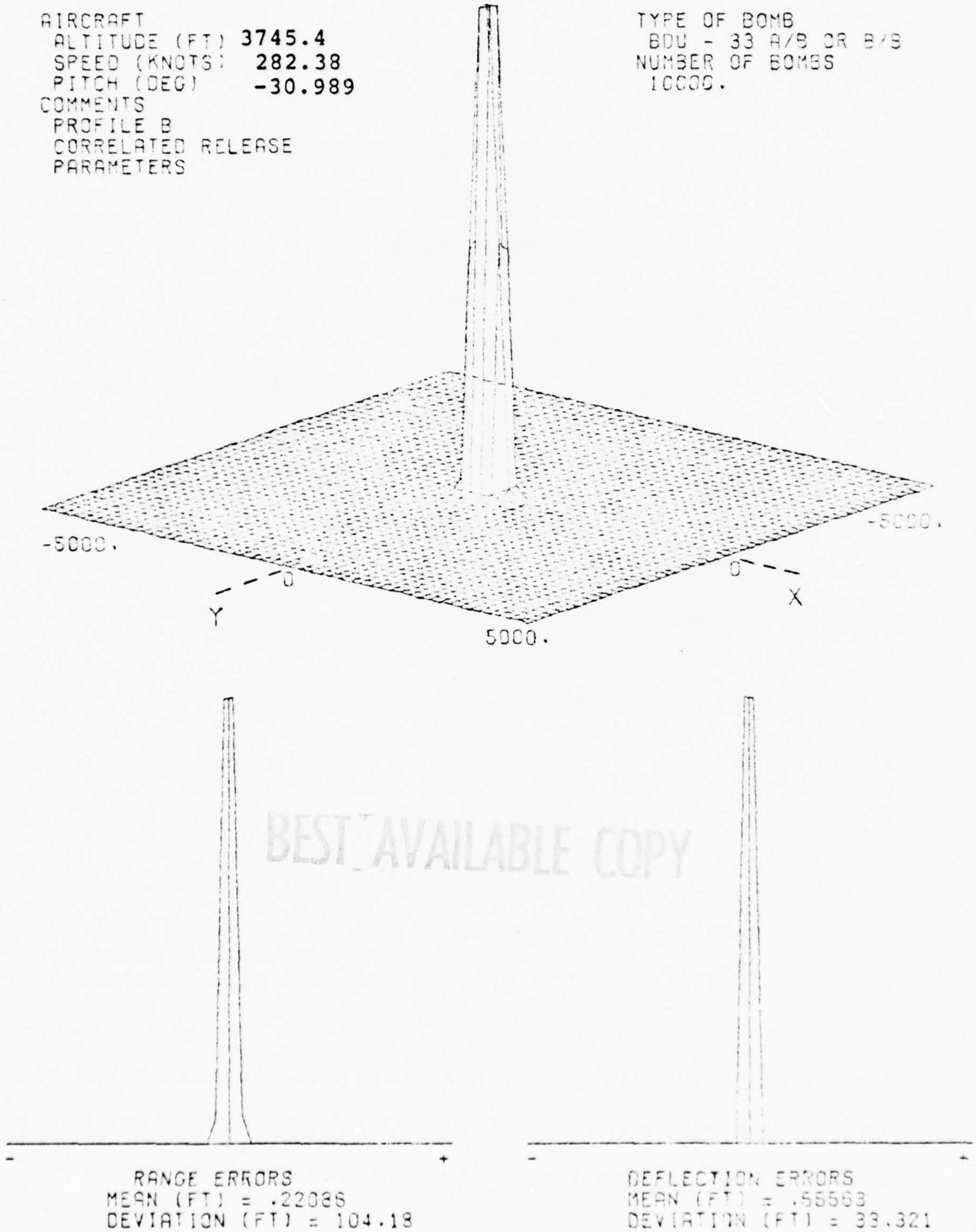


Figure K.2 Graph for Profile B Correlated Release Parameters

TABLE K.3

PROFILE B

ALTITUDE-LEAD UNCORRELATED

Input File

	μ	Alt	CAS	FPA	Lead	Hdg	Offset
Altitude:	3745.39	6756.13	0	0	0	0	0
Airspeed:	282.38	0	44.64	0	0	0	0
Pitch:	-30.99	0	0	1.98	-163.83	0	0
Lead:	8.83	0	0	-163.83	21535.68	0	0
Heading:	6.20	0	0	0	0	3.31	-115.28
Offset:	-212.78	0	0	0	0	-115.28	4913.00

Independence Check

Number of Cells: 16 χ^2 Critical Value: 16.919

Degrees of Freedom: 9 χ^2 Sample Value: 79.162

Statistical Analysis (10,000 Bombs)

	Min	Max	$\bar{x}$	s	χ^2 Crit	χ^2 Sample
X	-463.3	454.7	0.2	126.2	37.6520	19.7244
Y	-118.9	116.8	0.6	33.3	30.1440	24.9873
CE	0	463.9	109.1	74.7	--	--

F-test Results

	F-Crit	F-sample	Remarks
X	1.000	1.786	Reject null hypothesis
Y	1.000	7.279	Reject null hypothesis

Conclusion: The distribution is bivariate normal at the 95% level. The variance for correlated release parameters was less than that for independent release parameters.

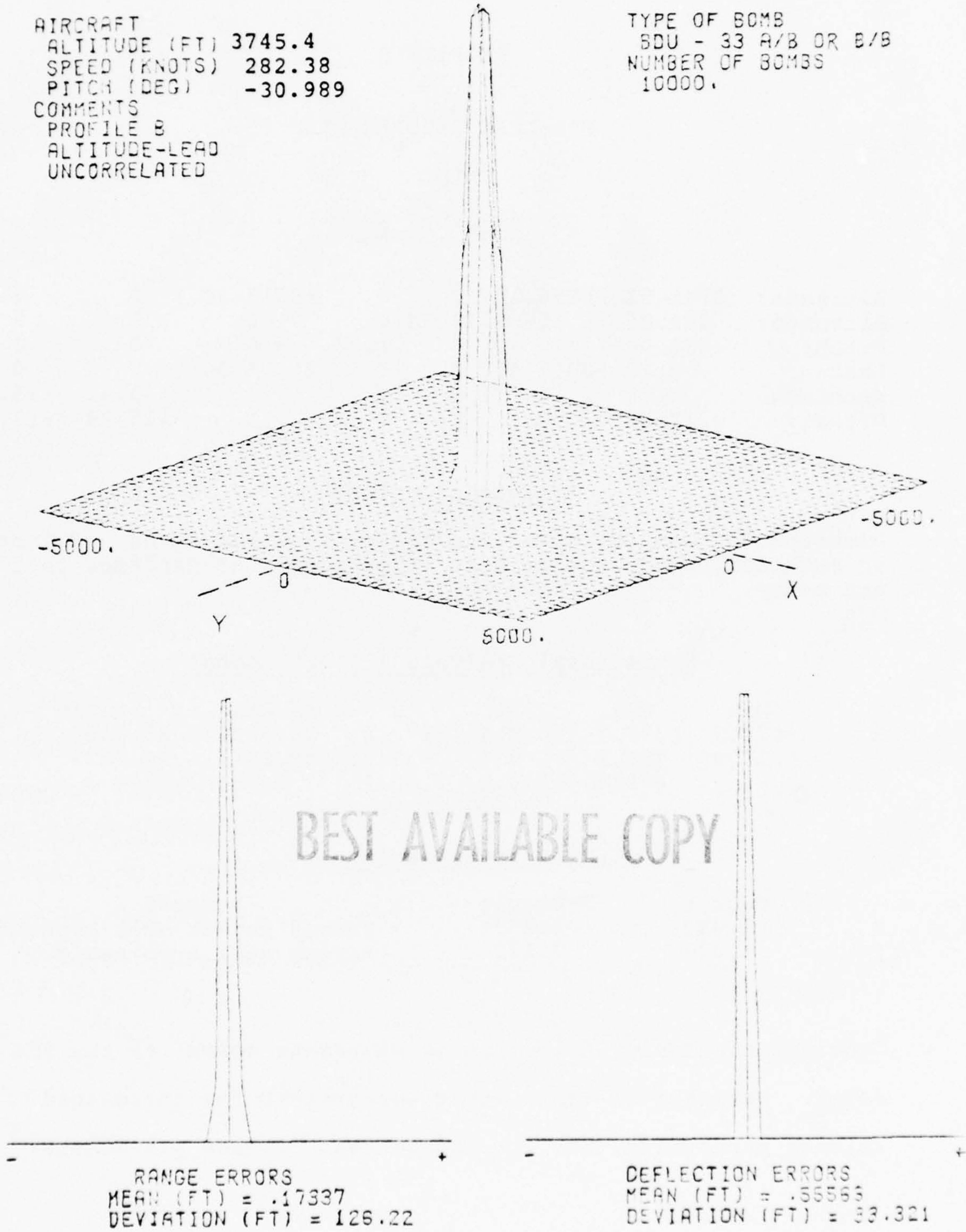


Figure K.3 Graph for Profile B Altitude-Lead Uncorrelated

TABLE K.4

PROFILE B

FPA-LEAD UNCORRELATED

Input File

Altitude:	3745.39	6756.13	0	0	-8017.40	0	0
Airspeed:	282.38	0	44.64	0	0	0	0
Pitch:	-30.99	0	0	1.98	0	0	0
Lead:	8.83	-8017.40	0	0	21535.68	0	0
Heading:	6.20	0	0	0	0	3.31	-115.28
Offset:	-212.78	0	0	0	0	-115.28	4913.00

Independence Check

Distribution was not bivariate and/or required number of impacts in each cell could not be met. Therefore, independence test was not made.

Statistical Analysis (10,000 Bombs)

	Min	Max	$\bar{x}$	s	χ^2 Crit	χ^2 Sample
X	-670.7	679.7	2.4	172.2	23.6850	20.3390
Y	-118.9	116.8	0.6	33.3	30.1440	24.9873
CE	0	679.9	144.0	100.1	--	--

F-test Results

	F-Crit	F-Sample	Remarks
X	1.000	0.959	Cannot reject null hypothesis
Y	1.000	7.279	Reject null hypothesis

Conclusion: The distribution is bivariate normal at the 95% level. Variance of range error was greater for correlated release parameters than for independent release parameters.

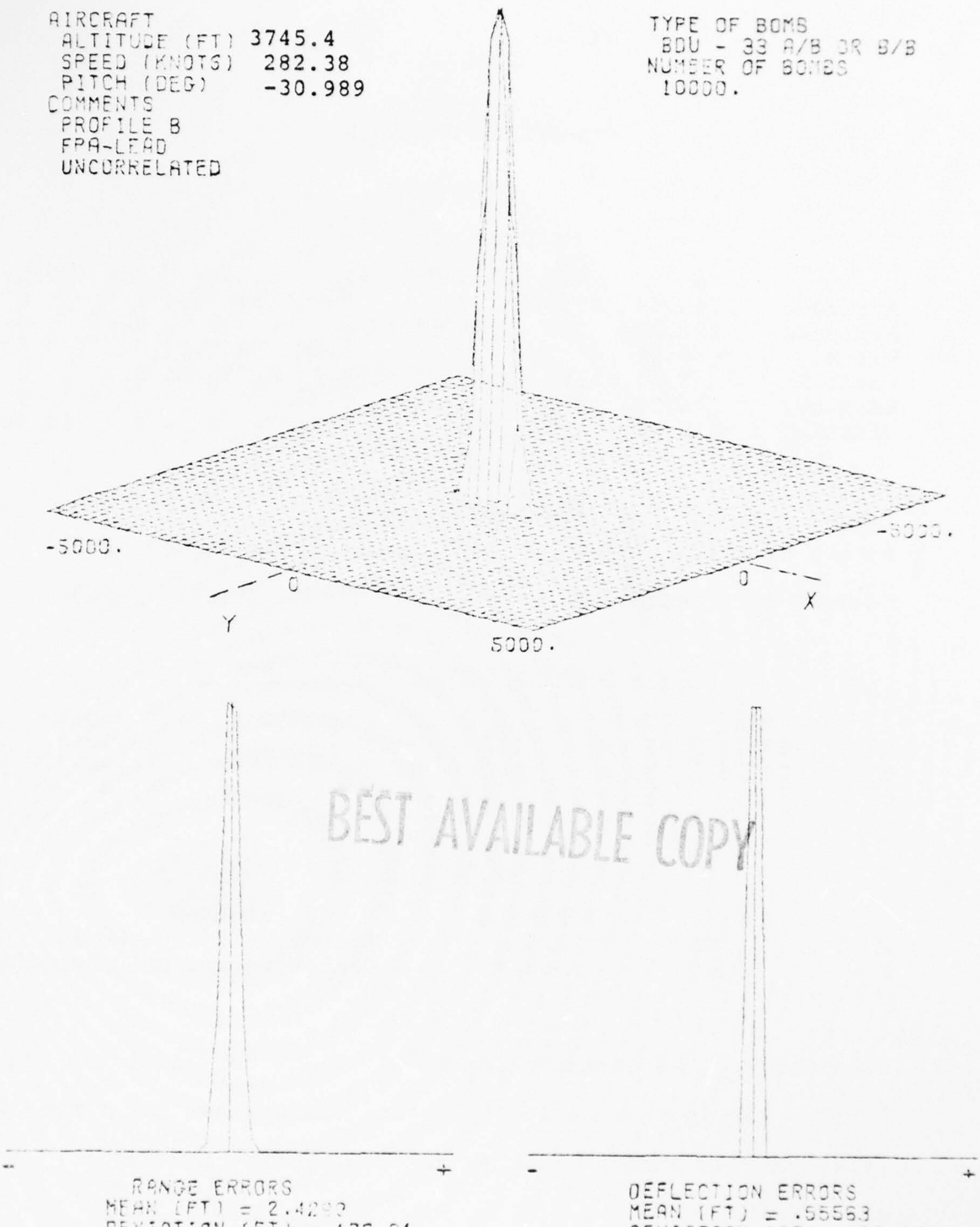


Figure K.4. Graph for Profile B FPA-Lead Uncorrelated

TABLE K.5

PROFILE B

HDG-OFFSET UNCORRELATED

Input File

	μ	Alt	CAS	FPA	Lead	Hdg	Offset
Altitude:	3745.39	6756.13	0	0	-8017.40	0	0
Airspeed:	282.38	0	44.64	0	0	0	0
Pitch:	-30.99	0	0	1.98	-163.83	0	0
Lead:	8.83	-8017.40	0	-163.83	21535.68	0	0
Heading:	6.20	0	0	0	0	3.31	0
Offset:	-212.78	0	0	0	0	0	4913.00

Independence Check

Number of Cells: 8 χ^2 Critical Value: 7.815

Degrees of Freedom: 3 χ^2 Sample Value: 3.600

Statistical Analysis (10,000 Bombs)

	Min	Max	$\bar{x}$	s	χ^2 Crit	χ^2 Sample
X	-375.9	363.0	0.2	104.2	48.3179	43.3914
Y	-356.5	399.8	0.8	98.7	19.6750	15.9457
CE	0	411.9	127.3	66.2	--	--

F-test Results

	F-Crit	F-Sample	Remarks
X	1.000	2.622	Reject null hypothesis
Y	1.000	0.829	Cannot reject null hypothesis

Conclusion: The distribution is bivariate normal at the 95% level. Variance of deflection error was greater for correlated release parameters than for independent release parameters.

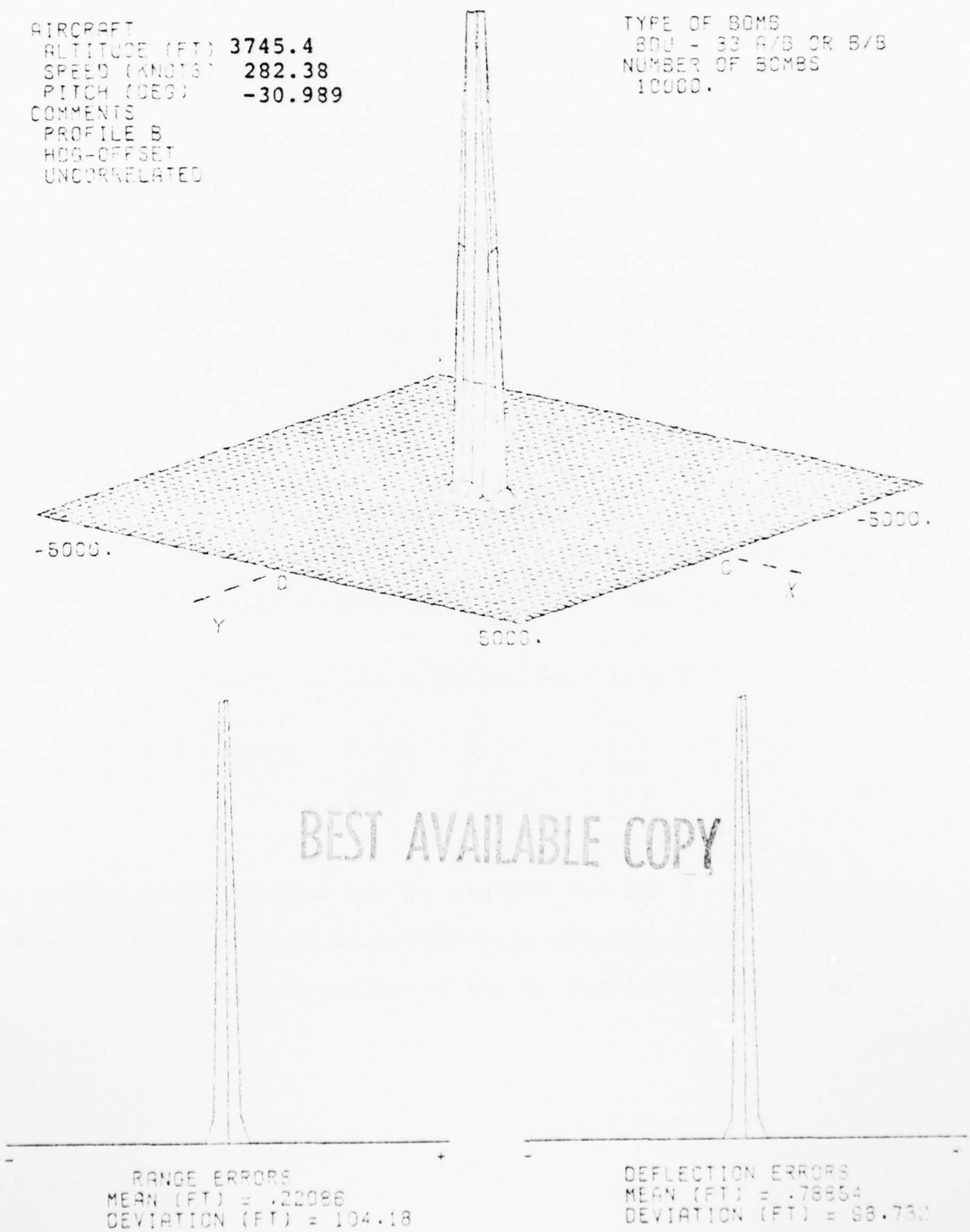


Figure K.5 Graph for Profile B Hdg-Offset Uncorrelated

TABLE K.6

PROFILE D

INDEPENDENT RELEASE PARAMETERS

Input Release Parameters

	μ	Min	Max	σ	Desired
Altitude:	3498.67	3336.00	3724.00	82.44	3498.67
Airspeed:	308.03	299.30	313.90	3.72	308.03
Pitch:	-31.31	-35.20	-26.60	2.48	-31.31
Lead:	90.11	-66.00	233.00	101.23	90.11
Heading:	6.91	0.23	13.72	4.06	6.91
Offset:	-177.39	-366.00	-56.00	84.53	-177.39

Independence Check

Number of Cells: 8 χ^2 Critical Value: 7.815

Degrees of Freedom: 3 χ^2 Sample Value: 20.559

Statistical Analysis (10,000 Bombs)

	Min	Max	$\bar{x}$	s	χ^2 Crit	χ^2 Sample
X	-479.1	613.0	7.9	160.8	(Failed	K-S Test)
Y	-375.0	398.4	-5.3	128.2	27.5870	81.8346
CE	0	645.5	183.7	92.9	--	--

Conclusion: X did not fit any of the distributions tested by SSIMFIT. Y is normally distributed at the 95% level. Therefore, the distribution is not bivariate normal.

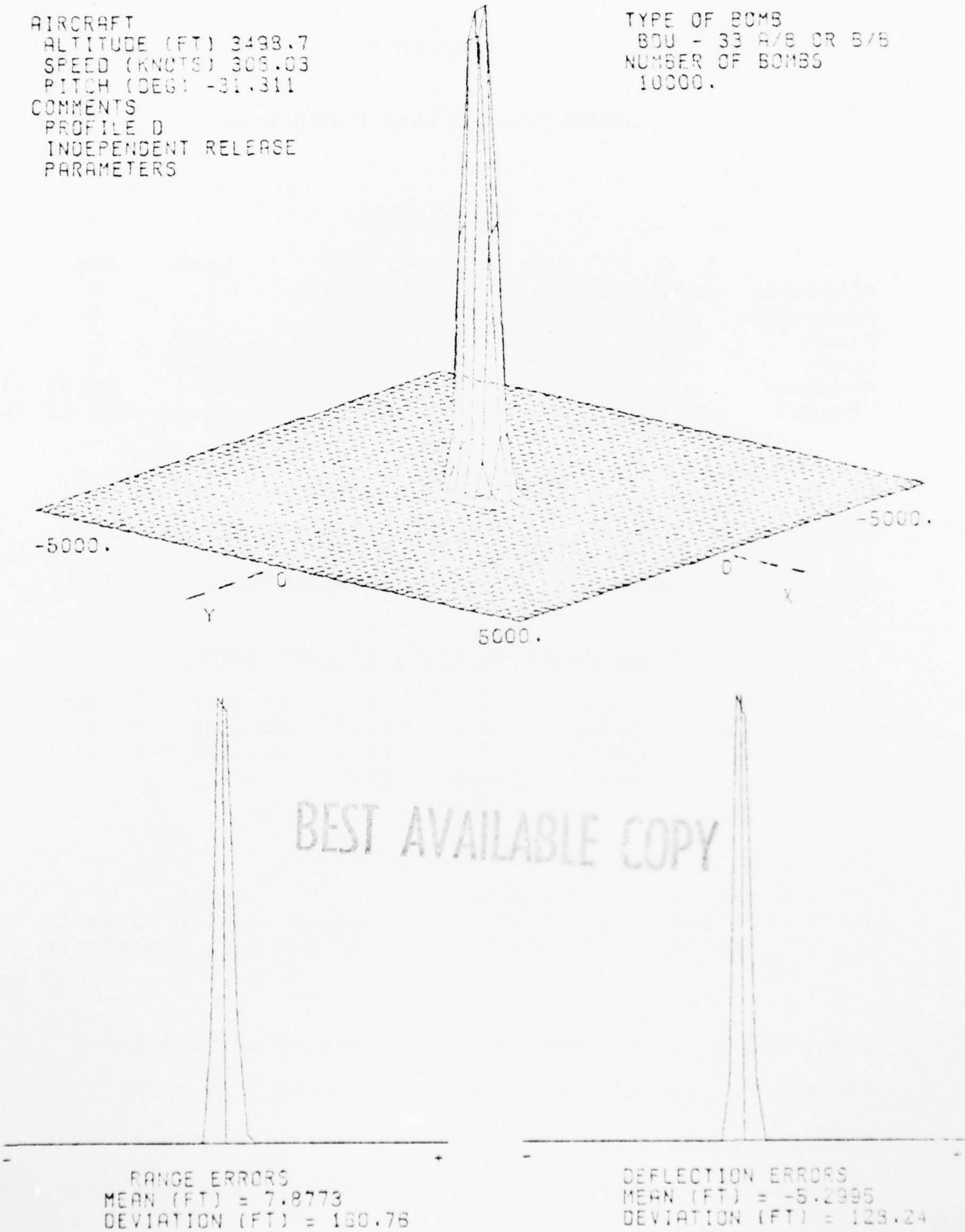


Figure K.6 Graph for Profile D Independent Release Parameters

TABLE K.7

PROFILE D

CORRELATED RELEASE PARAMETERS

Input File

	μ	Alt	CAS	FPA	Lead	Hdg	Offset
Altitude:	3498.67	6796.71	0	-116.31	0	0	0
Airspeed:	308.03	0	13.82	0	0	0	0
Pitch:	-31.31	-116.31	0	6.15	0	0	0
Lead:	90.11	0	0	0	10246.69	0	0
Heading:	6.91	0	0	0	0	16.53	-325.12
Offset:	-177.39	0	0	0	0	-325.12	7146.13

Independence Check

Number of Cells: 4 χ^2 Critical Value: 3.841

Degrees of Freedom: 1 χ^2 Sample Value: 15.761

Statistical Analysis (10,000 Bombs)

	Min	Max	$\bar{x}$	s	χ^2 Crit	χ^2 Sample
X	-620.5	592.0	-0.2	155.3	22.3620	22.3288
Y	-231.1	268.0	-0.1	65.1	(Failed K-S Test)	
CE	0	622.9	144.2	86.9	--	--

F-test Results

	F-Crit	F-Sample	Remarks
X	1.000	1.072	Reject null hypothesis
Y	1.000	3.882	Reject null hypothesis

Conclusion: X is normally distributed at the 95% level. Y did not fit any of the distributions tested by SSIMFIT. Therefore, the distribution is not bivariate normal. The variance for correlated release parameters was less than that for independent release parameters.

Figure K.7 Graph for Profile D Correlated Release Parameters

TABLE K.8

PROFILE D

ALT-FPA UNCORRELATED

Input File

	μ	Alt	CAS	FPA	Lead	Hdg	Offset
Altitude:	3498.67	6796.71	0	0	0	0	0
Airspeed:	308.03	0	13.82	0	0	0	0
Pitch:	-31.31	0	0	6.15	0	0	0
Lead:	90.11	0	0	0	10246.69	0	0
Heading:	6.91	0	0	0	0	16.53	-325.12
Offset:	-117.39	0	0	0	0	-325.12	7146.13

Independence Check

Number of Cells: 4 χ^2 Critical Value: 3.841

Degrees of Freedom: 1 χ^2 Sample Value: 8.450

Statistical Analysis (10,000 Bombs)

	Min	Max	$\bar{x}$	s	χ^2 Crit	χ^2 Sample
X	-706.3	777.8	3.7	192.8	19.6750	50.7190
Y	-230.0	303.5	0.3	67.3	(Failed K-S Test)	
CE	0	782.2	172.6	109.2	--	--

F-Test Results

	F-Crit	F-Sample	Remarks
X	1.000	.695	Cannot reject null hypothesis
Y	1.000	3.631	Reject null hypothesis

Conclusion: X is not normally distributed at the 95% level. Y did not fit any of the distributions tested by SSIMFIT. Therefore, the distribution is not bivariate normal. Variance of range error was greater for correlated release parameter than for independent release parameters.

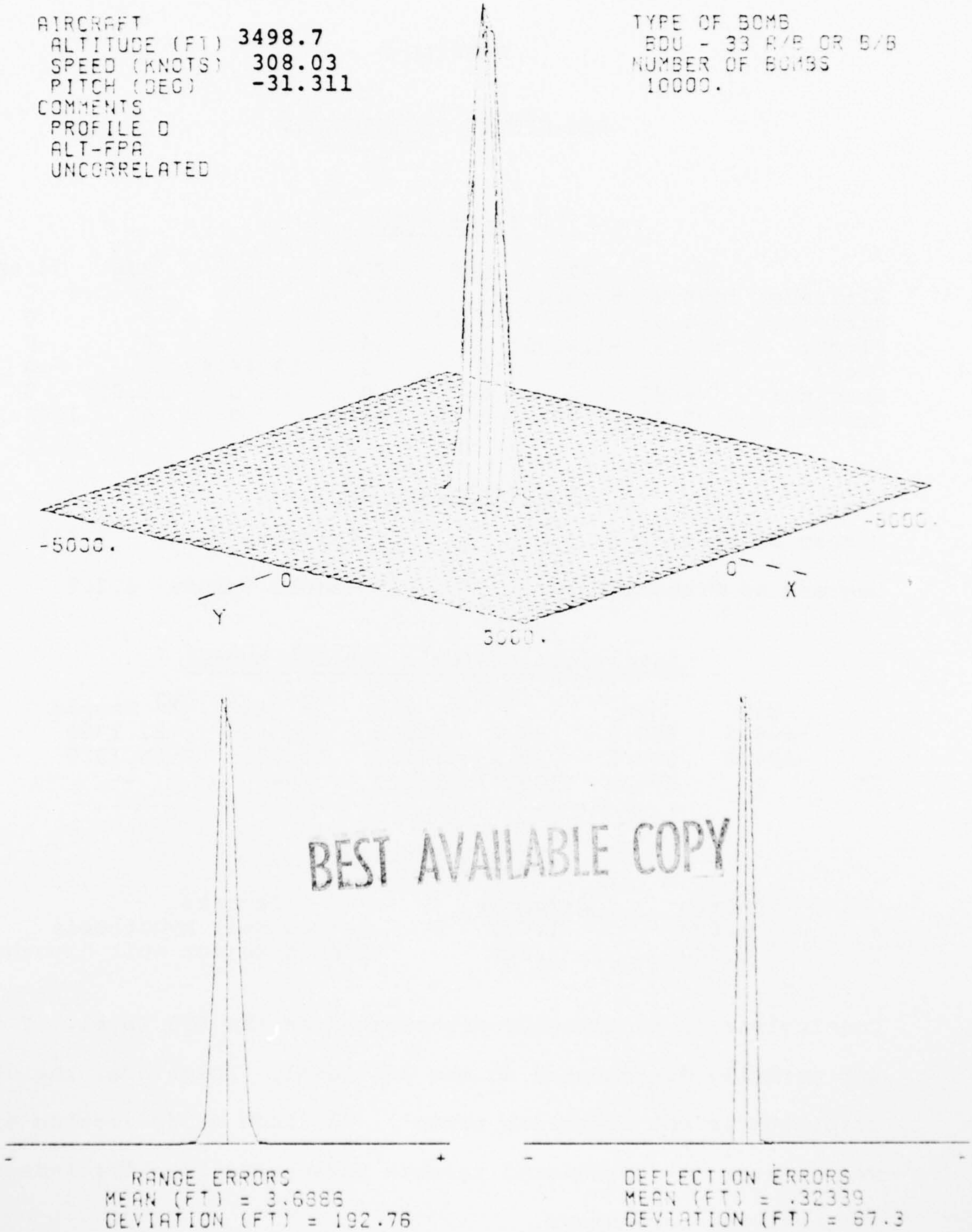


Figure K.8 Graph for Profile D Alt-FPA Uncorrelated

TABLE K.9

PROFILE D

HDG-OFFSET UNCORRELATED

Input File

	μ	Alt	CAS	FPA	Lead	Hdg	Offset
Altitude:	3498.67	6796.71	0	-116.31	0	0	0
Airspeed:	308.03	0	13.82	0	0	0	0
Pitch:	-31.31	-116.31	0	6.15	0	0	0
Lead:	90.11	0	0	0	10246.69	0	0
Heading:	6.91	0	0	0	0	16.53	0
Offset:	-177.39	0	0	0	0	0	7146.13

Independence Check

Number of Cells: 4 χ^2 Critical Value: 3.841

Degrees of Freedom: 1 χ^2 Sample Value: 3.208

Statistical Analysis (10,000 Bombs)

	Min	Max	$\bar{x}$	s	χ^2 Crit	χ^2 Sample
X	-620.5	592.0	-0.2	155.3	22.3620	22.3288
Y	-594.5	670.3	0.2	161.8	11.0700	55.4525
CE	0	674.9	199.6	102.3	--	--

F-test Results

	F-Crit	F-Sample	Remarks
X	1.000	1.072	Reject null hypothesis
Y	1.000	.628	Cannot reject null hypothesis

Conclusion: X is normally distributed at the 95% level. Y is not normally distributed at the 95% level. Therefore, the distribution is not bivariate normal. Variance of deflection error was greater for correlated release parameters than for independent release parameters.

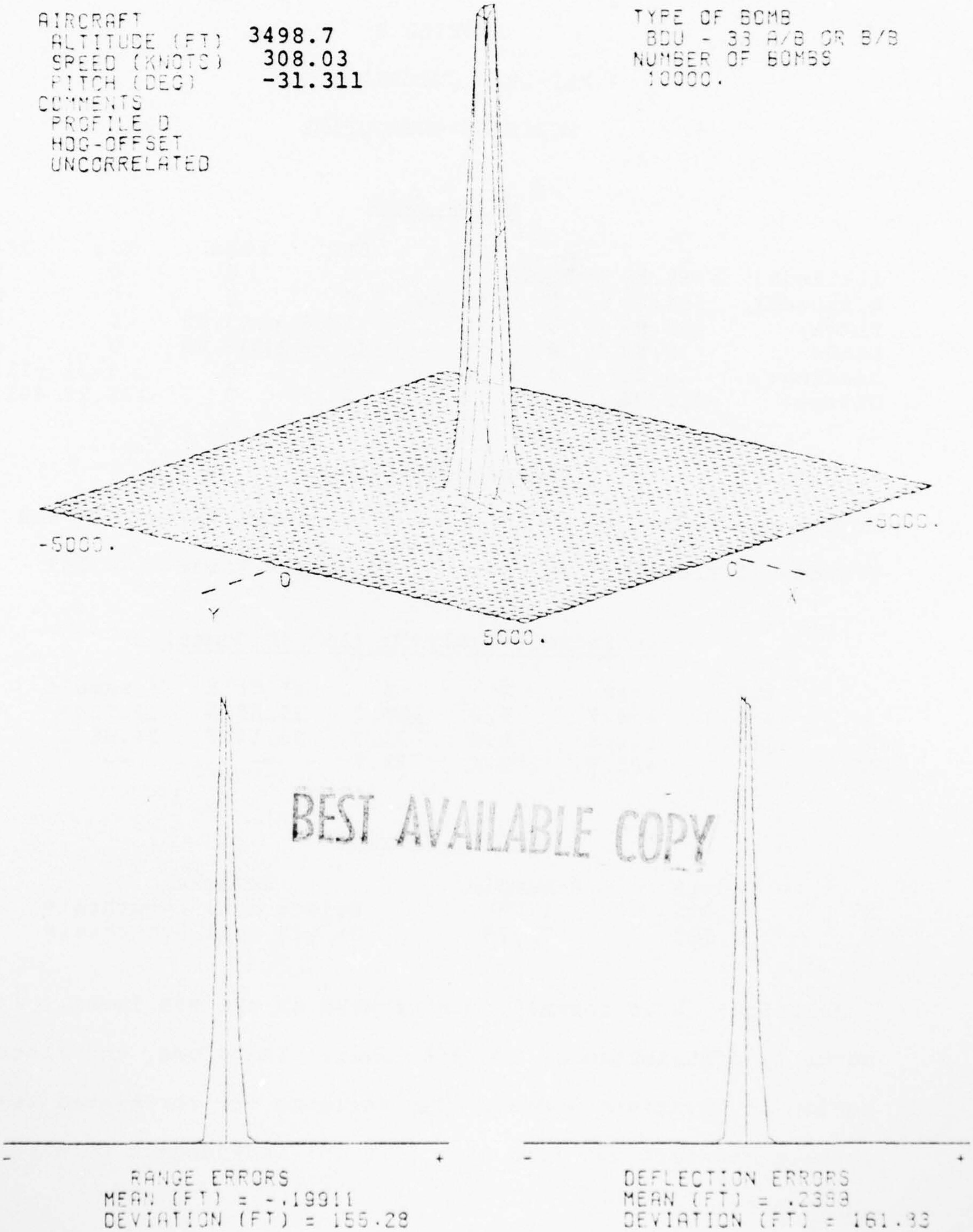


Figure K.9 Graph for Profile D Hdg-Offset Uncorrelated

TABLE K.10

PROFILE B

ALT-LEAD UNCORRELATED

MODIFIED SUBROUTINE

Input File

	μ	Alt	CAS	FPA	Lead	Hdg	Offset
Altitude:	3745.39	6756.13	0	0	0	0	0
Airspeed:	282.38	0	44.64	0	0	0	0
Pitch:	-30.99	0	0	1.98	-163.83	0	0
Lead:	8.83	0	0	-163.83	21535.68	0	0
Heading:	6.20	0	0	0	0	3.31	-115.28
Offset:	-212.78	0	0	0	0	-115.28	4913.00

Independence Check

Number of Cells: 16 χ^2 Critical Value: 16.919

Degrees of Freedom: 9 χ^2 Sample Value: 79.162

Statistical Analysis (10,000 Bombs)

	Min	Max	$\overline{x}$	s	χ^2 Crit	χ^2 Sample
X	-463.3	454.7	0.2	126.2	37.6520	19.7244
Y	-118.9	116.8	0.6	33.3	30.1440	24.9873
CE	0	463.9	109.1	71.7	--	--

F-test Results

	F-Crit	F-Sample	Remarks
X	1.000	1.786	Reject null hypothesis
Y	1.000	7.279	Reject null hypothesis

Conclusion: X is normally distributed at the 95% level. Y is normally distributed at the 95% level. Therefore, the distribution is bivariate normal. The variance for correlated release parameters was less than that for independent release parameters.

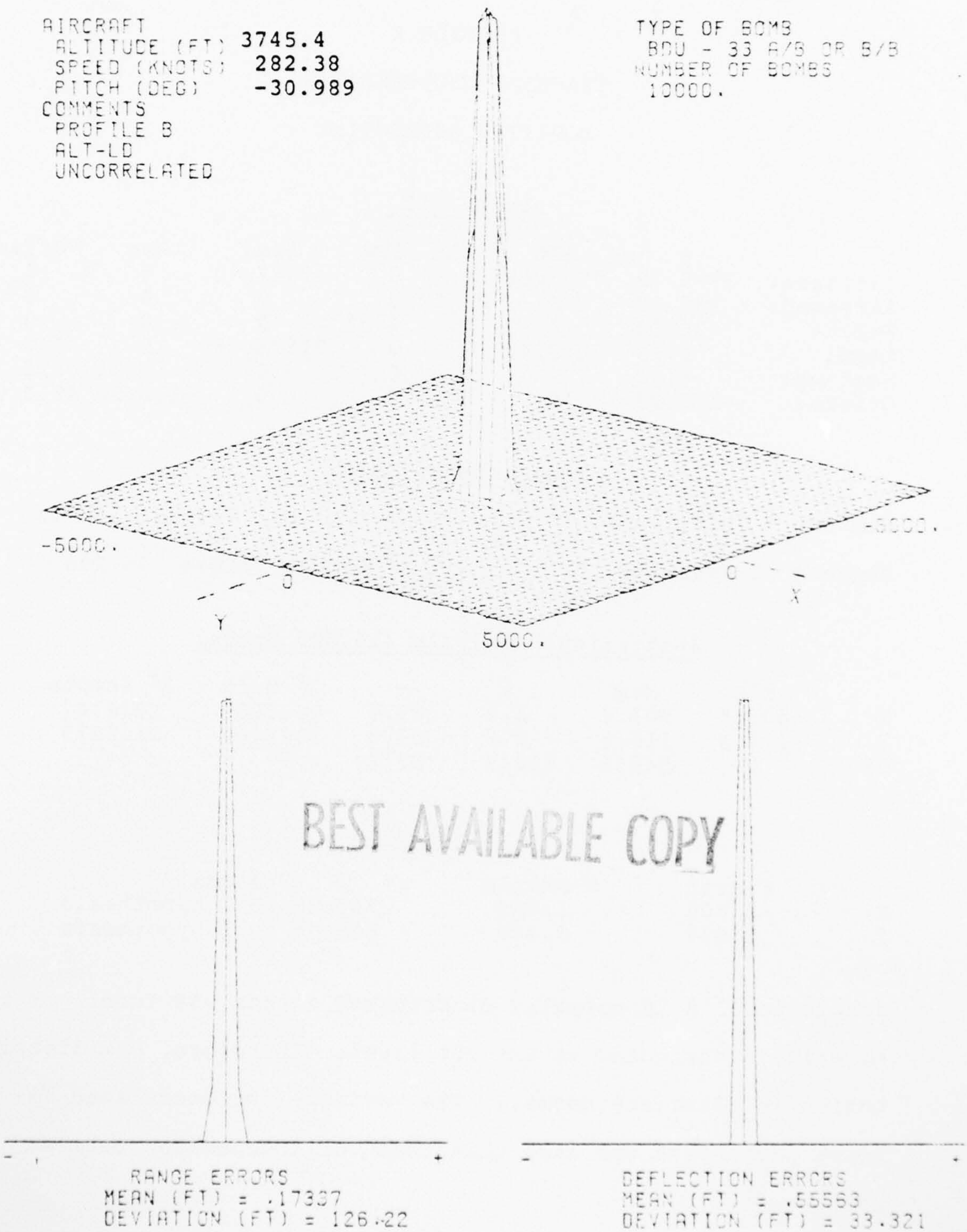


Figure K.10 Graph for Profile B Alt-Lead Uncorrelated Modified Subroutine

TABLE K.11

PROFILE B

FPA-LEAD UNCORRELATED

MODIFIED SUBROUTINE

Input File

	μ	Alt	CAS	FPA	Lead	Hdg	Offset
Altitude:	3745.39	6756.13	0	0	-8017.40	0	0
Airspeed:	282.38	0	44.64	0	0	0	0
Pitch:	-30.99	0	0	1.98	0	0	0
Lead:	8.83	-8017.40	0	0	21535.68	0	0
Heading:	6.20	0	0	0	0	3.31	-115.28
Offset:	-212.78	0	0	0	0	-115.28	4913.00

Independence Check

Number of Cells: 16 — χ^2 Critical Value: 16.919

Degrees of Freedom: 9 — χ^2 Sample Value: 96.298

Statistical Analysis (10,000 Bombs)

	Min	Max	$\bar{x}$	s	χ^2 Crit	χ^2 Sample
X	-535.6	503.7	1.4	139.6	26.2360	13.6747
Y	-118.9	116.8	0.6	33.3	30.1440	24.9873
CE	0	543.5	118.9	80.3	--	--

F-test Results

	F-crit	F-sample	Remarks
X	1.000	1.461	Reject null hypothesis
Y	1.000	7.279	Reject null hypothesis

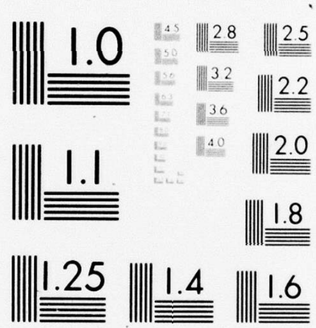
Conclusion: X is normally distributed at the 95% level. Y is normally distributed at the 95% level. Therefore, the distribution is bivariate normal. The variance for correlated release parameters was less than that for independent release parameters.

AD-A044 185 AIR FORCE INST OF TECH WRIGHT-PATTERSON AFB OHIO SCHO--ETC F/G 19/5
THE EFFECT OF RELEASE PARAMETER CORRELATIONS ON THE DISTRIBUTIO--ETC(U)
JUN 77 H A BROWN, M H CALLEN

UNCLASSIFIED AFIT-LSSR-25-77A NL

3 OF 3
ADA044185

END
DATE
FILMED
10-77
DDC



MICROCOPY RESOLUTION TEST CHART

NATIONAL BUREAU OF STANDARDS-1963-A

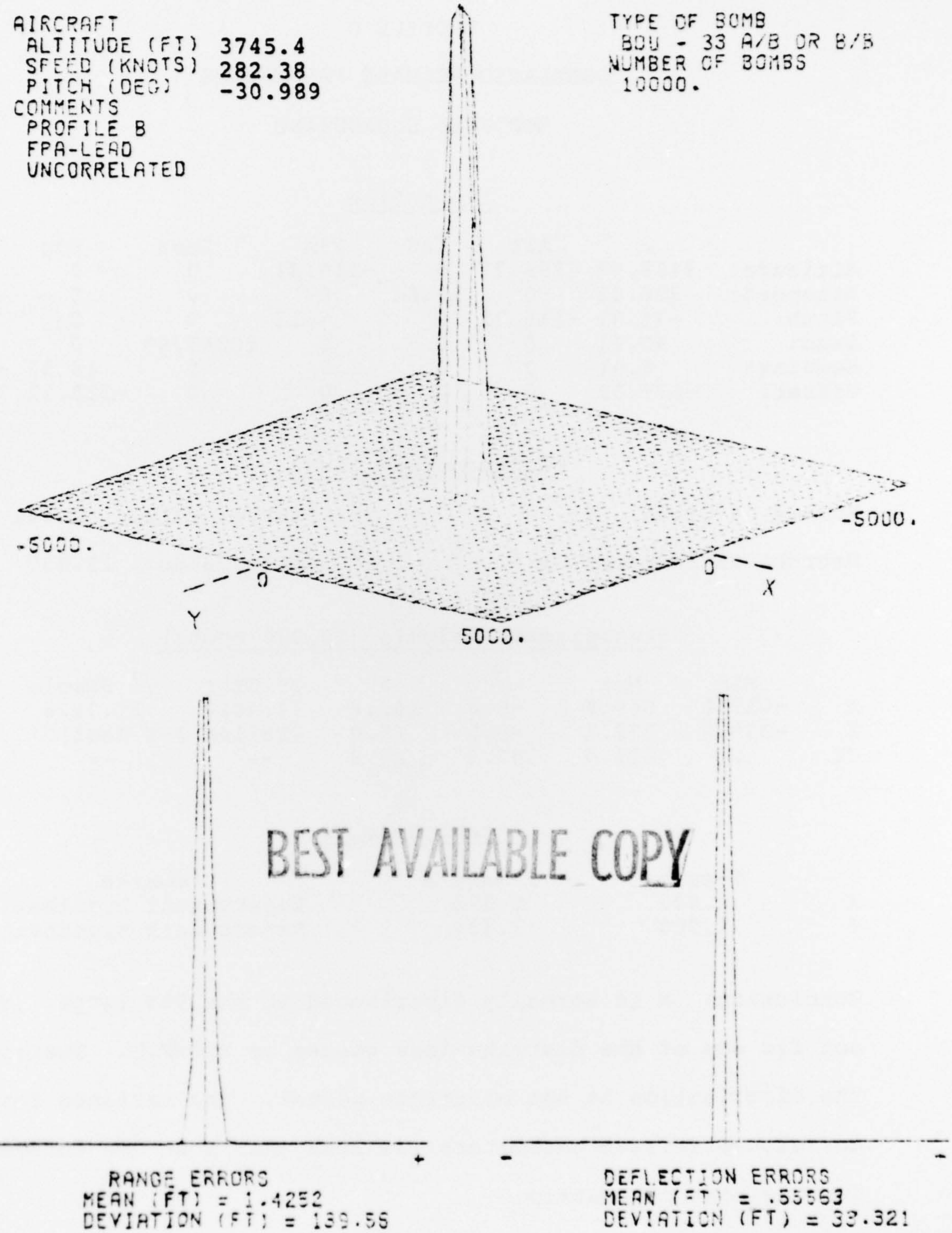


Figure K.11 Graph for Profile B FPA-Lead Uncorrelated Modified Subroutine

TABLE K.12

PROFILE D

CORRELATED RELEASE PARAMETERS

MODIFIED SUBROUTINE

Input File

	μ	Alt	CAS	FPA	Lead	Hdg	Offset
Altitude:	3498.67	6796.71	0	-116.31	0	0	0
Airspeed:	308.03	0	13.82	0	0	0	0
Pitch:	-31.31	-116.31	0	6.15	0	0	0
Lead:	90.11	0	0	0	10246.69	0	0
Heading:	6.91	0	0	0	0	16.53	-325.12
Offset:	-177.39	0	0	0	0	-325.12	7146.13

Independence Check

Number of Cells: 4 χ^2 Critical Value: 3.841

Degrees of Freedom: 1 χ^2 Sample Value: 15.035

Statistical Analysis (10,000 Bombs)

	Min	Max	$\bar{x}$	s	χ^2 Crit	χ^2 Sample
X	-619.1	569.5	-0.4	153.8	22.3620	21.7924
Y	-235.8	272.1	-0.2	65.0	(Failed K-S Test)	
CE	0	621.4	143.3	85.8	--	--

F-test Results

	F-crit	F-Sample	Remarks
X	1.000	1.092	Reject null hypothesis
Y	1.000	3.888	Reject null hypothesis

Conclusion: X is normally distributed at the 95% level. Y did not fit any of the distributions tested by SSIMFIT. Therefore, the distribution is not bivariate normal. The variance for correlated release parameters was less than that for independent release parameters.

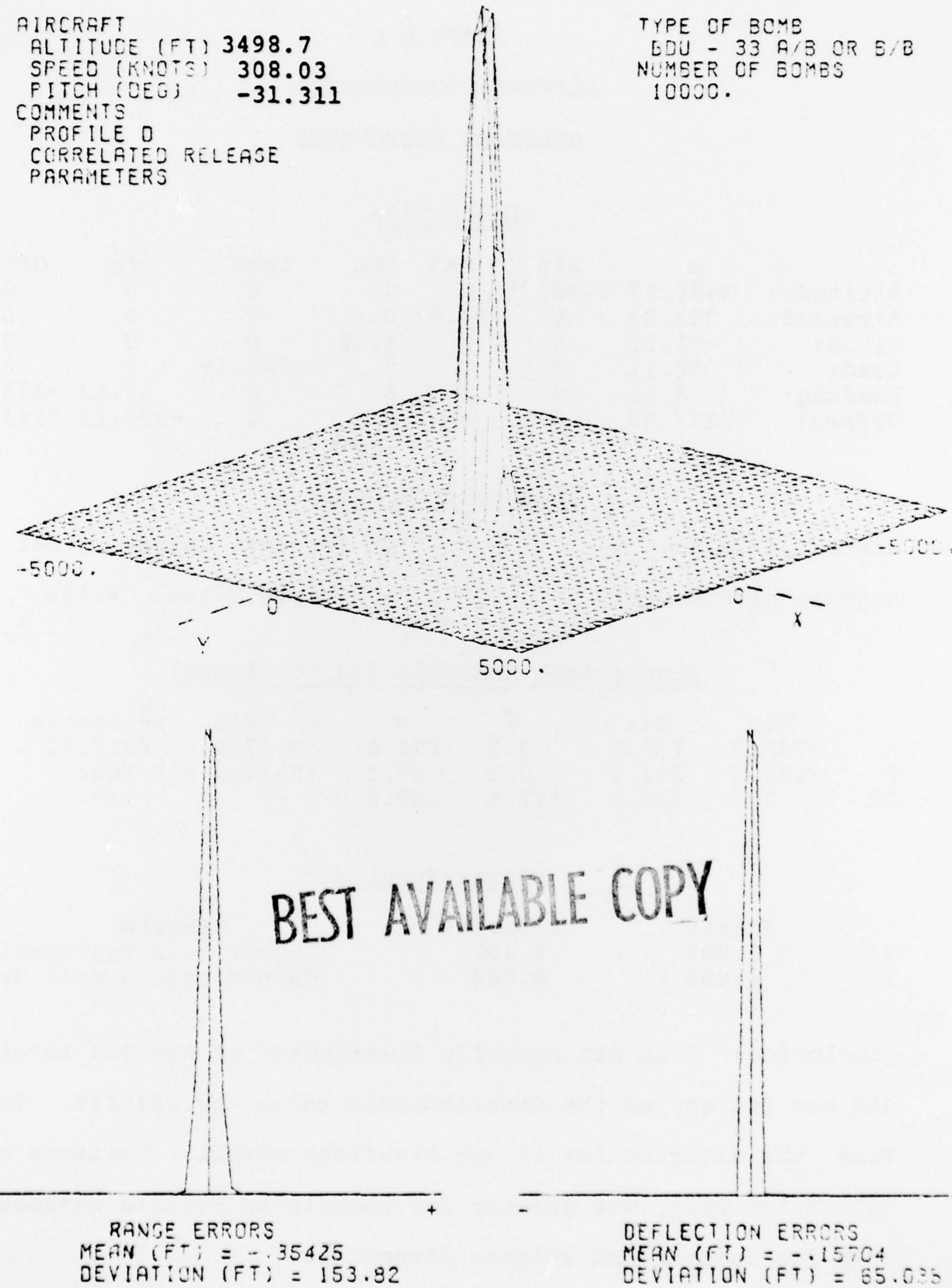


Figure K.12 **Graph for Profile D Correlated Release Parameters Modified Subroutine**

TABLE K.13

PROFILE D

ALT-FPA UNCORRELATED

MODIFIED SUBROUTINE

Input File

	μ	Alt	CAS	FPA	Lead	Hdg	Offset
Altitude:	3498.67	6796.71	0	0	0	0	0
Airspeed:	303.03	0	13.82	0	0	0	0
Pitch:	-31.31	0	0	6.15	0	0	0
Lead:	90.11	0	0	0	10246.69	0	0
Heading:	6.91	0	0	0	0	16.53	-325.12
Offset:	-177.39	0	0	0	0	-325.12	7146.13

Independence Check

Number of Cells: 4 χ^2 Critical Value: 3.841

Degrees of Freedom: 1 χ^2 Sample Value: 8.450

Statistical Analysis (10,000 Bombs)

	Min	Max	$\bar{x}$	s	χ^2 Crit	χ^2 Sample
X	-706.3	777.8	3.7	192.8	19.6750	50.7190
Y	-230.0	303.5	0.3	67.3	(Failed K-S Test)	
CE	0	782.2	172.6	109.2	--	--

F-test Results

	F-crit	F-sample	Remarks
X	1.000	1.092	Reject null hypothesis
Y	1.000	0.628	Cannot reject null hypothesis

Conclusion: X is not normally distributed at the 95% level. Y did not fit any of the distributions tested by SSIMFIT. Therefore, the distribution is not bivariate normal. Variance of deflection error was greater for correlated release parameters than for independent release parameters.

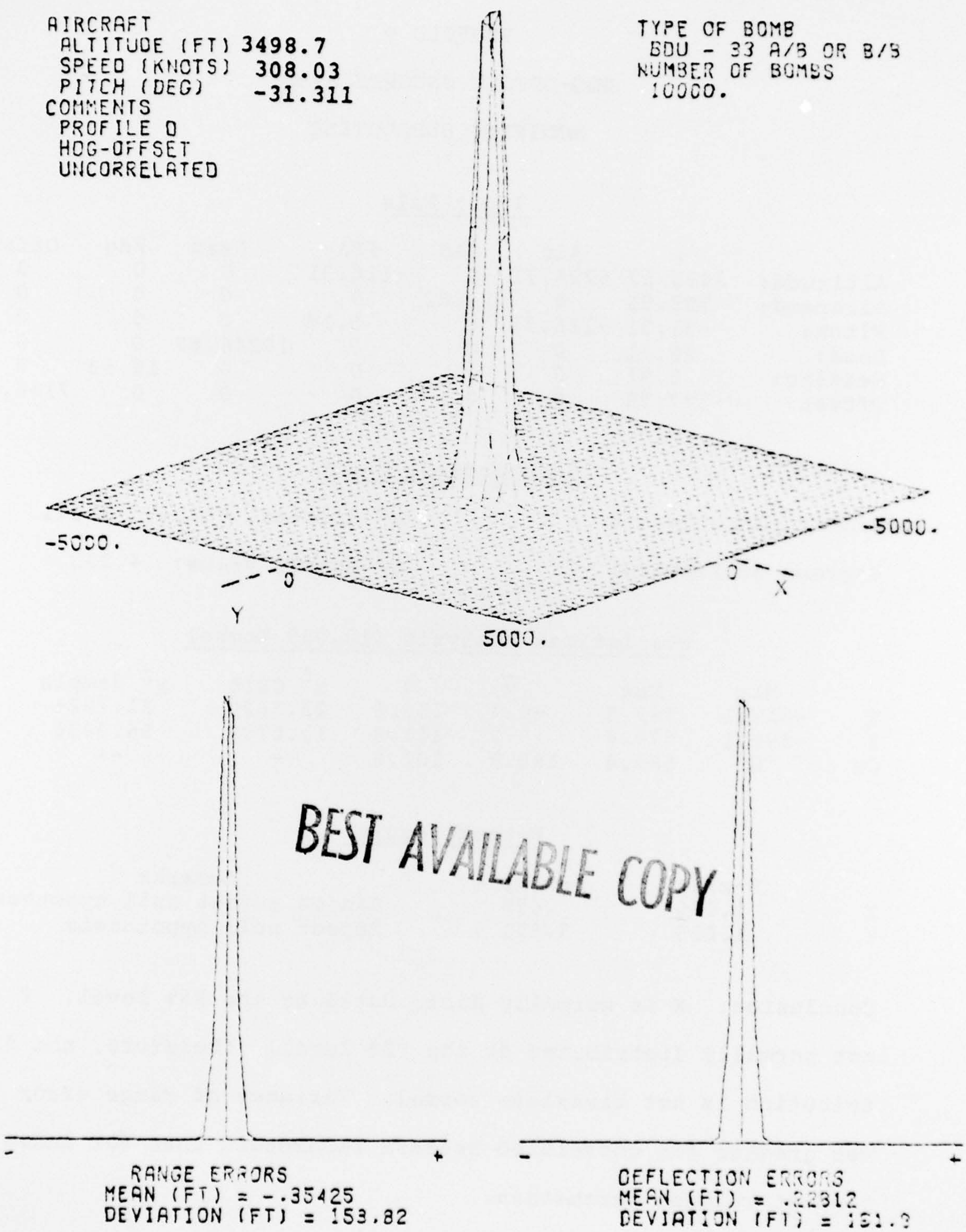


Figure K.13 Graph for Profile D Alt-FPA Uncorrelated Modified Subroutine

TABLE K.14

PROFILE D

HDG-OFFSET UNCORRELATED

MODIFIED SUBROUTINE

Input File

	μ	Alt	CAS	FPA	Lead	Hdg	Offset
Altitude:	3498.67	6796.71	0	-116.31	0	0	0
Airspeed:	308.03	0	13.82	0	0	0	0
Pitch:	-31.31	-116.31	0	6.15	0	0	0
Lead:	90.11	0	0	0	10246.69	0	0
Heading:	6.91	0	0	0	0	16.53	0
Offset:	-177.39	0	0	0	0	0	7146.13

Independence Check

Number of Cells: 4 χ^2 Critical Value: 3.841

Degrees of Freedom: 1 χ^2 Sample Value: 4.033

Statistical Analysis (10,000 Bombs)

	Min	Max	$\bar{x}$	s	χ^2 Crit	χ^2 Sample
X	-619.1	569.5	-0.4	153.8	22.3620	21.7924
Y	-599.1	674.4	0.2	161.8	11.0700	56.3036
CE	0	680.4	198.8	101.6	--	--

F-test Results

	F-crit	F-sample	Remarks
X	1.000	.695	Cannot reject null hypothesis
Y	1.000	3.631	Reject null hypothesis

Conclusion: X is normally distributed at the 95% level. Y is not normally distributed at the 95% level. Therefore, the distribution is not bivariate normal. Variance of range error was greater for correlated release parameters than for independent release parameters.

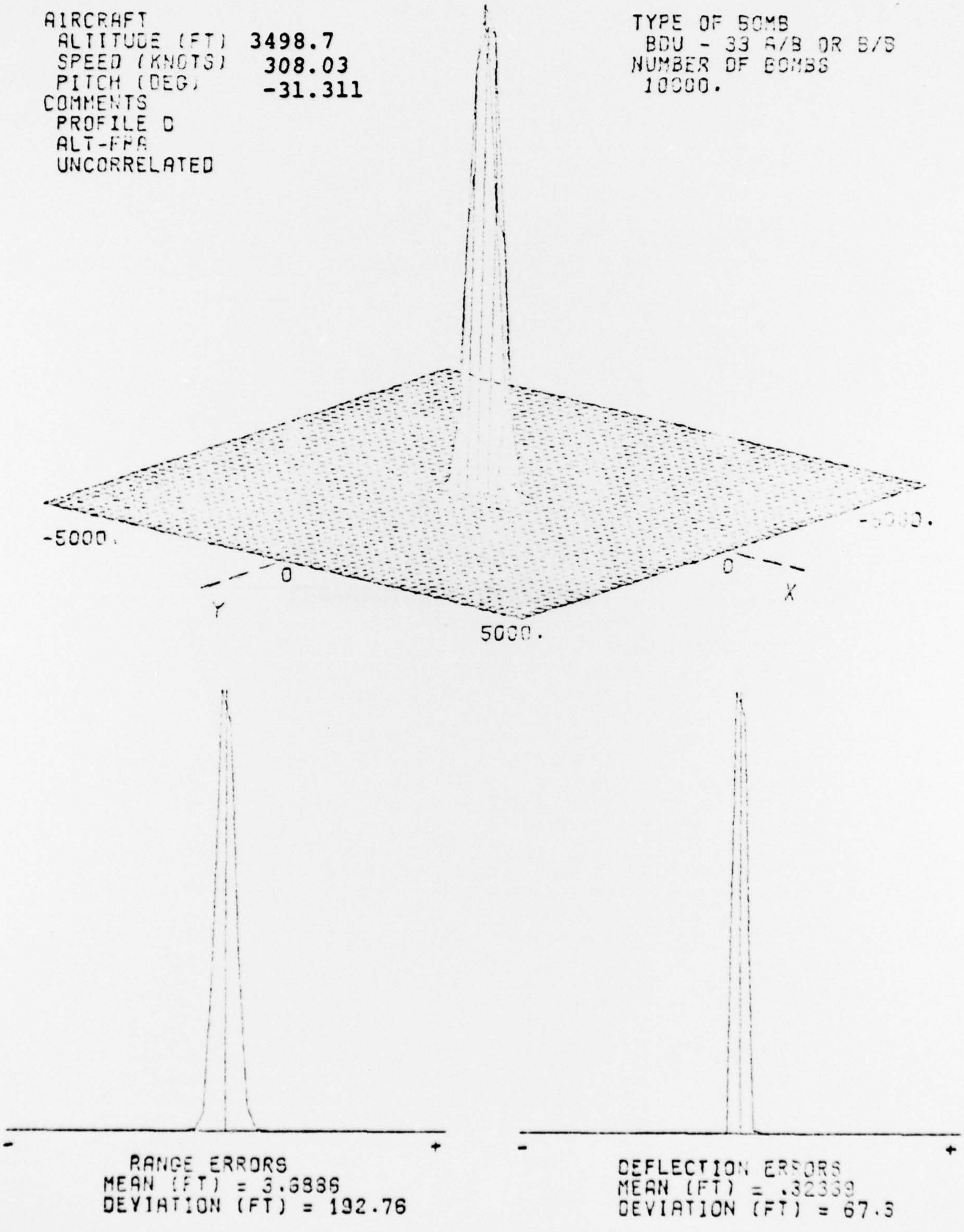


Figure K.14 Graph for Profile D Hdg-Offset Uncorrelated Modified Subroutine

SELECTED BIBLIOGRAPHY

A. REFERENCES CITED

1. Appel, Lieutenant Colonel Bernard. "Bombing Accuracy in a Combat Environment," Air University Review, XXVI (July-August, 1975), pp. 38-52.

2. Bent, Dale H., C. Hadlai Hull, Jean G. Jenkins, Norman H. Nie, and Karin Steinbrenner. Statistical Package for the Social Sciences. 2nd ed. McGraw-Hill, Inc. 1975.

3. Berry, Captain William G., and Captain Leon W. Laugginger. "A Computer Simulation of Release Parameter Effects Upon Bomb Impact Distributions." Unpublished Master's Thesis. SLSR 18-75A, AFIT/SL, Wright-Patterson AFB, Ohio, 1975.

4. Biehle, Major Kenneth H. "An Analysis of the Effect of Air Crew Factors on B-52 Bombing Results." Unpublished Master's Thesis. GSA/SM/73-4, AFIT, Wright-Patterson AFB, Ohio, 1973.

5. Chamberlain, Captain Brian. "A Preliminary Empirical Analysis of Bomb Impact Data." Unpublished technical report. AFIT/SL, Wright-Patterson AFB, Ohio, 1971.

6. Clark, Charles T. and Lawrence L. Schkade. Statistical Analysis for Administrative Decisions. 2nd ed. Cincinnati: South-western Publishing Company, 1974.

7. Conover, W. J. Practical Nonparametric Statistics. New York: John Wiley & Sons Inc., 1971.

8. Downs, Captain David B., and Captain Rolf E. Forseth. "The Effect of Changes In Release Parameters Upon Bomb Impact Distributions: A Computer Model." Unpublished Master's Thesis. SLSR 18-76A, AFIT/SL, Wright-Patterson AFB, Ohio, 1976.

9. Emory, C. William. Business Research Methods. Homewood, Illinois: Richard D. Irwin, Inc., 1976.

10. Fairchild Republic Company. A-10A Airplane Bombing Accuracy Evaluation. Fairchild Republic Engineering Flight Test Memorandum Report FT160RFB07, Part 2, 29 October 1975.

11. Goodman, Stanley, and Kenneth Hewlitt. "Frankford Aircraft Capabilities Test (FACT)." Unpublished technical research report. FA-TR-74001, ARMCOM, Rock Island, Illinois, 1974.

12. Hackford, Captain Richard H., Jr. "Analysis of Piloted Weapon Delivery: F-4E Aerial Gunnery." Unpublished Master's Thesis. AFIT, Wright-Patterson AFB, Ohio, 1972.

13. Hovde, Captain Robert John. "Analysis of Piloted Weapon Delivery: F-4C Dive Bombing." Unpublished Master's Thesis. GGC/EE/72-6, AFIT/SE, Wright-Patterson AFB, Ohio, 1972.

14. Lawrence, Captain Frederick P. Instructor of Logistics Management, School of Systems and Logistics, Air Force Institute of Technology (AU), Wright-Patterson AFB, Ohio. Lecture Notes. 20 October 1976.

15. ________. Instructor of Logistics Management, School of Systems and Logistics, Air Force Institute of Technology (AU), Wright-Patterson AFB, Ohio. Lecture Notes. 22 October 1976.

16. ________. Instructor of Logistics Management, School of Systems and Logistics, Air Force Institute of Technology (AU), Wright-Patterson AFB, Ohio. Lecture Handout, Subject: Kolmogorov-Smirnov Test. 17 November 1976.

17. Murdoch, David C. Analytic Geometry with an Introduction to Vectors and Matrices. New York: John Wiley & Sons, Inc., 1966.

18. National Bureau of Standards. Handbook of Mathematical Functions. Washington: Government Printing Office, 1964.

19. Naylor, Thomas H. Computer Simulation Experiments with Models of Economic Systems. New York: John Wiley & Sons, Inc., 1971.

20. Naylor, Thomas H. and others. Computer Simulation Techniques. New York: John Wiley & Sons, Inc., 1966.

21. Schroeter, G. "Impact Precision for Automatic Bomb-Release Systems." Unpublished technical report. AD906324, FTD/TDBDR, Wright-Patterson AFB, Ohio, 1969.

22. Siegel, Sidney. Nonparametric Statistics for the Behavioral Sciences. New York: McGraw-Hill Book Company, 1956.

23. U.S. Department of the Air Force, the Army, and the Navy. Joint Munitions Effectiveness Manuals. AF Regulation 8-4. Government Printing Office, Washington, D.C., 1969.

24. Valstar, Jacob E. "Dynamic Weapon Delivery Error Analysis Program, Volume I." Unpublished technical report. AD915063, Naval Development Center, Attn: Warminster, PA 1973.

25. Wonnacott, Thomas H., and Ronald J. Wonnacott. Introductory Statistics for Business and Economics. New York: John Wiley & Sons, Inc., 1972.

26. Zambo, Major Leslie J. Instructor of Logistics Management, School of Systems and Logistics, Air Force Institute of Technology (AU), Wright-Patterson AFB, Ohio. Lecture Notes. 15 September 1976.

B. RELATED SOURCES

Anderson, T. W. An Introduction to Multivariate Statistical Analysis. New York: John Wiley & Sons, Inc., 1958.

Brown, Thomas H., and Commander John C. Clinton. "Effects of Release Interval for Stick Bombing on Probability of Kill." Unpublished technical report. AD918829, Naval Weapons Center, China Lake, CA 1974.

Richardson, Captain Michael E. "An Analysis of the Effect of Release Condition Errors on Bomb Support as Associated with the F-111A and F-111E Trail and Range Bombing Modes." Unpublished Master's Thesis. SLSR 46-71B, AFIT/SL, Wright-Patterson AFB, OH 1971.

Walker, Norman K. "Evaluation of ZITA/ADT Performance Measuring Technique, ZITA/ADT Test on Six A-37B Pilots and a Comparison with Dive Bombing Errors." Unpublished final report. OAS-TR-74-5, USA Material Systems Analysis Agency, Aberdeen Proving Ground MD 1974.